Ingeneri, P. J. The present status of multi-sensory aids in the
1947 mathematics classrooms...

BOSTON UNIVERSITY

SCHOOL OF EDUCATION

Thesis

THE PRESENT STATUS OF MULTI-SENSORY AIDS IN THE MATHEMATICS CLASSROOMS AND LABORATORIES OF MASSACHUSETTS SECONDARY SCHOOLS

Submitted by

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(B.S. in Ed. Boston University School of Education, 1946)

In partial fulfillment of requirements for
the degree of Master of Education

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May 7, 1947
28042

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CHAPTER I

INTRODUCTION

THE PROBLEM

It is the primary objective of this thesis to report on the present status of multi-sensory aids in the mathematics classrooms and laboratories of Massachusetts secondary schools. To what extent are these classrooms and laboratories equipped with multi-sensory aids and what use is made of some of the aids on hand? This is the main question to be answered by the present study.

There are two additional questions which the study will attempt to answer:

1. What multi-sensory aids are available for the teaching of mathematics, independent of their present status?
2. Where can these multi-sensory aids be obtained?

It is to be hoped that the answers to these questions will contribute to the improvement and enrichment of mathematics instruction in secondary schools.

<u>The Case for Multi-Sensory Aids</u>

Multi-sensory aids! What will they think of next? This might be the natural reaction of one who has attempted to keep abreast of developments in this birth-of-atomic energy age. Why do we need a term "multi-sensory aids" when we have heard so much and done so much with visual aids, audio-visual aids, visual sensory aids, and the like? Is this term necessary? If so, what does it tell us that the other terms do not?

Perhaps it would be well at this point to review briefly the development of the above-mentioned terms, and thus conceivably find justification for the adoption of the new term "multi-sensory" aids.

In the early stages of the development of scientific aids to learning, it was believed by some that the visual sense was of paramount importance in the education process. Dent[1] tells us that some enthusiastic supporters of this view went so far as to say that 90 per cent of all one learns is learned through the eye. Davis[2] on the other hand, tells us that some advocates of visual aids believe

1. Ellsworth C. Dent, <u>Audio-Visual Handbook</u>. The Society for Visual Education. Chicago. 1937, p.1.

2. Robert A. Davis, <u>Psychology of Learning</u>. McGraw-Hill Book Company, Inc. New York. 1935, p. 357.

that at least 40 per cent of conceptual learning may be attributed to the sense of vision, while others have been more conservative.

Is an experiment which purports to isolate and indicate the definite contribution made to the learning process by any single sense possible? The writer does not think it very likely. The obvious difficulty of conducting a controlled experiment along these lines would make the reliability of the results questionable. Investigations into the relative effects of different modes of presenting material to be learned have been conducted by Freeman[1], Lacy[2], O'Brien[3], and Russell[4] among others. These investigators came to conclusions such as these:

1. No one mode of presentation appears best for all individuals.

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1. F.N.Freeman, Visual Education. University of Chicago Press. Chicago, 1924.

2. John V. Lacy, "The Relative Value of Motion Pictures as an Educational Agency". Teachers College Record, 20:452-465, 1919.

3. F.J.O'Brien, "A Quantitative Investigation of the Effect of Mode of Presentation upon the Process of Learning". American Journal of Psychology, 32:249-283, 1921.

4. R.D. Russell, "A Comparison of Two Methods of Learning", Journal of Educational Research, 18:235-238, 1928.

2. No one mode of presentation is best for the same individual when different materials are used.
3. Learning by different methods is usually influenced by the age and maturity of pupils.
4. The comparative effectiveness of visual aids as distinguished from lecture and other verbal methods of classroom instruction depends upon the nature of the instruction and the character of the pupil's experience.

These findings alone will suffice to show that any statement to the effect that 90 per cent or 40 per cent of what is learned is learned through the visual sense is not, and conceivably cannot be backed by objective evidence.

In time, workers in the field began to consider the possibility that the claims made for the visual sense may have been a bit extravagent. Perhaps the other senses also had a significant contribution to make to the learning process. Perhaps a clear and complete impression of material to be learned can be had only by the harmonious operation of all the senses.

Evidence to support this view was not lacking. Dent[1] speaks on this subject as follows:

> "It was found that the sense of touch aided materially in giving the correct concepts of objects, specimens, and models. The sense of smell became important in many situations. In other cases, taste came to the rescue. And the sometimes unadorning ear is being recognized as highly important to learning."

In addition, the inventive genius of our nation had given us phonographs, sound and silent films, film-strips, and a host of other scientific developments which, by application of a little teaching "genius" could be made to contribute greatly to the educative process.

As a result of the evolving theories on the learning process, together with the development of scientific aids to learning, we find the appearance of such terms as visual-sensory aids, auditory aids, audio-visual aids, etc., etc.. More recently, the theories on the learning process have developed to the point where educators, on the whole, would agree that all the senses at one time or another, or all at the same time contribute to more effective and permanent learning. Williams[2] has this to say in support of this view:

1.Dent, Ibid. p. 4

2.L.A.Williams, <u>Secondary Schools for American Youth</u>. American Book Company. New York. 1944, p. 430.

"Learning can be made more complete and more permanent under the principle that variety as well as amount of sensory stimulation improves learning".

Davis[1] makes this contribution:

"It is usually best to use many avenues of approach in learning. In some cases, it is desirable to experience the situation by simultaneous stimulation of many sense avenues........the principle of utilizing all avenues of approach to learning illustrates the law of association and the development of perceptions. The greater the number of sense organs stimulated, the larger the number of associations formed".

In "Fundamentals of Educational Psychology"[2] we find the following statement:

"One avenue of sense presentation is about as good as another in the case of adults. With children, as many of the senses should be utilized as possible. In teaching numbers, the children should be made to feel, see, hear, manipulate and write. Learning through motor activity is very economical".

If we accept the statements of these educators and psychologists, but still carry further the analogy of a name for each aid which stresses some sensory avenue, would this not result in unnecessary cluttering of the literature with such terms as: olfactory aids, gustatory aids, tactual aids, etc.? While seemingly bordering on

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1. Robert A. Davis, Psychology of Learning. McGraw-Hill Book Company, Inc. New York. 1935, p.355.

2. Ira M. Gast and Harley C. Skinner, Fundamentals of Educational Psychology. Benj. H. Sanborn & Co. Chicago. 1929, p. 204.

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the facetious, such would have to be the case unless some one term could be used to cover all types of aids to learning.

The National Council of Teachers of Mathematics settled on the word "multi-sensory" to be used in connection with all types of learning aids. This word used in connection with aids to learning appeared for the first time in the literature dealing with theories of education in the Eighteenth Yearbook of the Council[1].

The writer feels that the term "multi-sensory" is completely in line with current educational thought as it applies to scientific aids to learning. While it has made its initial appearance in connection with the field of Mathematics, it is applicable to all fields of learning. It is sincerely believed by this writer that much more is to be gained from an investigation of the status of multi-sensory aids in mathematics classrooms and laboratories, than from an investigation of visual aids or audio-visual aids alone. Not only will it make for a more nearly complete and accurate picture of just what aids are being

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1. The National Council of Teachers of Mathematics, Eighteenth Yearbook, Multi-Sensory Aids in the Teaching of Mathematics. Bureau of Publications, Teachers College. Columbia University, New York. 1945, p.l.

used, and the extent to which some of these aids are used; but it is also to be hoped that teachers seeing the term multi-sensory for the first time may be themselves stimulated into investigating the significance and implications of this "new" word.

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CHAPTER II

MULTI-SENSORY AIDS AVAILABLE FOR THE TEACHING OF MATHEMATICS AS REVEALED BY THE LITERATURE

Previous Studies in the Field

<u>None on multi-sensory aids</u>.

In a previous chapter, it was mentioned that the term "multi-sensory" aids in connection with mathematics appeared for the first time in the literature in the Eighteenth Yearbook of the National Council of Teachers of Mathematics. This may explain why there did not appear in the literature any study investigating the status of these aids in the mathematics classrooms and laboratories of Massachusetts publich high schools.

<u>Studies on sub-types</u>.

Numerous studies have been made of sub-types of multi-sensory aids: visual aids, audio-visual aids, etc.. Since the main objective of the present study is to investigate the status of multi-sensory aids in mathematics classrooms and laboratories of Massachusetts high schools, it would appear that any study of aids to teaching in fields other than mathematics, and places other than Massachusetts would make no significant contribution to the present study.

The literature did reveal one study by Chapman[1]

1. Leland Hildreth Chapman, "The Present Status of Visual Aids in the Secondary Schools of Massachusetts". Unpublished Master's thesis, Boston University School of Education, Boston, 1938, 91 pp.

which is worthy of mention. The study dealt with the status of visual aids in the secondary schools of Massachusetts. Chapman investigated the status of visual aids in all departments, but the writer is particularly interested in what he has to say with respect to the mathematics department:

"It is generally conceded that this department uses very few visual aids and table 16-A bears this out".

Table 16-A, as presented in the Chapman study, preceded by a brief table defining groups I, II, III, IV and V follows:

	Group I	Group II	Group III	Group IV	Group V
Enrollment	Less than 250	250 - 499	501 - 899	900 - 1399	1400 and more
Number of schools reporting.	35	43	38	37	35

Table 16-A Percentages of schools using visual aids in the mathematics department during the first semester of 1937 - 1938.

Visual Aids	Group I	Group II	Group III	Group IV	Group V	Total
Journeys. . . .	6	2	0	0	0	2
Motion Pictures						
Silent.	0	0	0	3	3	1
Sound	0	0	0	0	0	0
Still Pictorial Material. . . .						
Stereographs. .	0	2	0	9	9	2
Glass Slides. .	0	5	0	0	0	2
Film Strips . .	0	0	0	0	0	0
Opaque Material	0	2	8	20	20	9
Total	1	2	1	5	5	2

Chapman concludes his report on the status of visual aids in the mathematics departments with these words:

> "With the exception of opaque material, practically no use is made of visual aids and the opaque material itself is hardly worth consideration. So few schools did anything with visual aids in this department that the average number of times aids were used would be of no interest".

The above statement, is justified on the basis of the results obtained in the Chapman study. Would the same statement be justified today? Has there been any change in the status of these aids? If there has, what kind of change has it been? It may prove interesting to determine the answers to some of these questions by comparing the results as determined for a given aid in the present study with the results reported for the same aid in the Chapman study. This will be done in the chapter dealing

with statistical analysis and interpretation of check list results.

The stated purpose of the present study is to report on the status of certain specified multi-sensory aids for the teaching of mathematics. In view of this, it is obviously not within the realm of the study to define each specified aid exhaustively. However, in the pages that follow, an attempt will be made to justify briefly the use of the term "multi-sensory" in connection with certain aids, and to make suggestions for the use of the aids listed in the check list in mathematics classrooms and laboratories.

Multi-sensory aids as defined in Chapter I are those which make for the utilization of more than one avenue of sense perception in the educative process. If, in the suggestions which follow in connection with the use of different aids, it can be shown that the pupils' eyes, ears, and hands will be coordinated in the educative process, the use of the term "multi-sensory" in connection with the aid will be considered justified.

In the Appendix are listed selected sources for all the aids investigated in the present study. Much valuable information about the uses of certain aids can be obtained by writing to the appropriate "source". The selected bibliography also to be found in the Appendix contains references for readings which illustrate what is being done by many progressive teachers of mathematics in the field of multi-sensory aids.

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Blackboards and Blackboard Equipment

Blackboards.

If there is any single aid which might be considered indespensable in a mathematics classroom, that aid is a blackboard. The blackboard is so much a part of every classroom that it is difficult to conceive of any classroom, and a mathematics classroom in particular, without one. Teachers turn almost instinctively to the blackboard for writing notices, examination questions, outlines and assignments, and for drawing graphs, diagrams, sketches, etc.

In the ideal situation, what is written on the board is in full clear view of all the pupils. Since what appears on the blackboard often remains there for some time, and in some cases may be considered an example for the pupil to follow, it is worthwhile to remember that "example is a much more powerful teaching device than precept".

As for squared blackboards, it is usually sufficient to have one section of the plain blackboard ruled and outlined in pale green or yellowpaint, with heavier lines for every fifth or tenth space. Such squared blackboards make for greater accuracy in graph work. This type of blackboard is used extensively in algebra, general mathematics, and trigonometry. In algebra, graphs are a vital part of the work dealing with concepts of variation and dependence or function. Solutions of linear and quadratic equations are

made more meaningful through the media of squared blackboards and graphs.

In a course in trigonometry, the squared blackboard is used extensively in connection with the work on trigonometric functions. Statistics also comes in for its share of graph work for which this aid is well nigh indispensable.

There is little need of going any further with illustrations of this type. It can be seen that wherever graph work contributes to the educative process, effective use can be made of a squared blackboard. An accurate graph contributes much to the effectiveness of certain phases of mathematical instruction, and it cannot be disputed that a squared blackboard contributes much to greater accuracy.

<u>Spherical blackboard</u>.

A spherical blackboard is extremely useful in helping pupils to visualize spherical angles and polygons in solid geometry. It helps greatly to dispel some of the difficulties which pupils have in visualizing the relation of lines, points, planes, and spherical surfaces to each other, and to the sphere itself. In connection with mathematical applications it might be used to illustrate and trace great circles. It might be used to illustrate certain types of map projection and the representation of a curved surface on a flat surface. These and many others are the uses to which the imaginative and resourceful teacher can put spherical blackboards, an

aid made prominent by its absence in so many mathematics classrooms, much to the detriment of teacher and pupils.

Blackboard rules, compasses, and protractors.

These tools should be among the equipment of every mathematics classroom. It is desirable that each mathematics classroom be equipped with at least two of each of these tools. These could be used by teacher and pupils in blackboard drawing and construction work.

The writer has seen mathematics teachers capable of admirably accurate free-hand drawing. In the writer's experience, however, this has been the exception rather than the rule. The added time needed in using rulers, compasses, and protractors for blackboard drawings and construction is more than made up for by the accuracy of the impression which such drawings and construction convey. Use of these tools also aids in elimination of wrong impressions which might result from drawings which are not so accurate. A mathematics teacher might tell a class that every point on the circumference of a circle is equidistant from the center and then make a free-hand drawing which would belie such a statement. Along this same line, a teacher might tell pupils that in equilateral triangles, all sides are of equal length and all angles are equal, and then make a free-hand drawing of a triangle which would do little to support such a definition. Teachers and pupils of mathematics could add many more examples similiar to those given above. In contrast of these examples, accurate drawings made with ruler, compass, and

protractor could be easily recognized from and reconciled with the teacher's definition.

Blackboard equipment if used frequently by the teacher might prove helpful in getting pupils to use more effectively their smaller counterparts of these tools. If this is not done, pupil drawings will approximate the sloppiness of the teacher's. Rulers, compasses, and protractors are such simple tools that a little practice will soon bring complete mastery. By way of practice pupils might be asked to measure angles present everywhere about them in the classroom; in the woodwork, in the supports of their desks, in the windows, on the doors, and a multitude of other places. The dimensions of the blackboard, the bulletin board, the floor of the room, the tops of desks, might be determined by the pupils and checked by the teacher or by the pupils themselves. Pupils might be asked to experiment on drawings or designs made by using compasses alone. Teacher and pupils will marvel at the beauty and variety of the drawings and design which will be produced. By example and by suggestion the teacher can do much to make effective use of rulers, compasses, and protractors by her pupils the rule rather than the exception.

Pantographs.

The pantograph is a device which has long been ignored in mathematics classrooms and laboratories. Extremely satisfactory pantographs can be made from strips of cardboard with brass paper fasteners for joints. A wealth of geometry can be discovered from the use of the pantograph. Trisection of certain angles can be accomplished by using this device which can also be used as parallel rulers.

Pantographs can be used to great advantage in the family of theorems dealing with similarity and parallel lines. It will not only facilitate instruction in this area, but make it more interesting for pupils at the same time. The pupils can not only see and hear about this device, but can manipulate it themselves in the solution of different problems. Thus, the use of another sense is enrolled in the educative process.

Colored chalk.

Few persons would question the statement that a picture in color has a greater eye-appeal than one in black and white. The tremendous popularity of technicolor films and the Sunday comics would appear to support such a statement. In view of this greater appeal which color lends to a picture, why could not, and should not greater use be made of color in the teaching of certain mathematical subjects, particularly geometry.

An idea of what can be done with color in geometry can

be obtained from an early book on plane geometry by Oliver Byrne[1]. In this book, angles, triangles, and other figures are identified by their color, ie; the red angle, or the blue triangle rather than angle A B C or triangle P D Q. Where color is used to identify a figure, recognition is almost instantaneous. Where letter symbols are used, on the other hand, the angle, triangle, or other figure under consideration is not always readily identified by all pupils.

Colored chalk may be used to illustrate the relationship among families of theorems. It can be used to great advantage in theormes dealing with congruency and similarity by coloring corresponding parts the same color. It can be used in certain definitions. For example, in defining "a line", the instructor can color in a rectangular block on the black board and then draw a rectangular block of a different color touching the first block. The boundary between the two blocks may be defined as a line. By this method, the pupils will more readily graps the concept that a line has no thickness. These are but a few of the many uses to which color can be put in geometry.

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1. Oliver Byrne, "The First Six Books of the Elements of Euclid in Which Coloured Diagrams and Symbols are Used Instead of Letters for the Greater Ease of Learners". London, 1847.

Still Pictorial Material

Maps

An aid seldom utilized in mathematics classes is maps. They can be used in algebra classes in connection with problems of time, rate, and distance. In algebra and geometry, they can be used to clarify the concept of scale and proportion. In geometry and trigonometry maps can be used in connection with problems in triangulation and navigation by land, sea, and air.

There is a wealth of mathematics to be uncovered in a discussion of the actual construction of maps; in showing the different ways in which a round surface can be projected onto a flat surface. Pupils themselves might be asked to attempt the construction of maps. This would put to good use the pupils' creative spirit, and the desire to work with their hands as well as with their "eyes" and "ears".

Graphs

Graphs are used extensively in mathematics courses, particularly algebra, trigonometry, and general mathematics. In some instances, it is an effective aid in reducing the symbolism of numbers and makes comparisons and contrasts of different groups of related numbers easier. A column of figures is not nearly as effective or interesting as a good graph which requires little explanation and tells its story at a glance. The construction of bar graphs, "pie" graphs, line graphs, and pictorial graphs will prove fascinating as

well as educational to many pupils.

A discussion of graphs in mathematics need not be restricted to statistical graphs alone. In algebra, they can be used to great advantage in making the solution of linear and quadratic equations more readily understandable to pupils particularly where complex roots are involved. The concepts of function, slope, and scale also lend themselves readily to graphical illustration and representation.

It is difficult to conceive of a course in trigonometry which does not make extensive use of graphs. How much more effective could instruction in trigonometry functions of angles be made if, in addition to tabular representation of the numerical values of the sine and tangent for different angles, the graphical representation of these values could also be supplied. This and other uses of graphs can make the work of both teacher and pupils more interesting and educationally effective.

Diagrams and posters.

Diagrams and posters, just as other still pictorial materials, are valuable aids to teaching because they are easily obtained and constructed and can be presented at crucial points in the instruction. Just as with other still pictorial materials, diagrams and colorful posters make for effective bulletin board displays.

A diagram may be a simple straight or broken line graph, or a complicated drawing illustrating the use or construction of a piece of equipment used in a mathematics classroom or laboratory. A carefully drawn and labeled diagram can prove effective in illustrating the concept of scale, particularly if pupils are asked to construct diagrams drawn to scale. The reverse process, or the construction of a device or piece of equipment from the instruction given in a diagram can also be used to advantage in mathematics courses, particularly courses in vocational or shop mathematics.

The billboards so common to the American scene may be said to be the prototype of the classroom poster. An ideal size for a classroom poster is 28 by 22 inches. In the construction of a poster, one may utilize charts, graphs, pictures, diagrams or any still pictorial material which helps to tell a story at a glance.

Pupils with a liking for drawing will need little urging to construct posters around a given mathematical topic. These posters may be black and white or in color. They may be drawn by the pupils, or may be constructed by mounting pictures, diagrams, cartoons, graphs, charts, and other materials. It is desirable that these posters be of uniform size if they are to be stored for future use in files provided for that purpose. Pupils with a liking for photography might take pictures and mount them in poster form.

Colorful, well constructed, and pertinent posters can be used effectively in bulletin board displays and exhibits. The following is an illustration from the Eighteenth Yearbook of the National Council of Teachers of Mathematics[1], of the use of posters in an exhibit. The part of the exhibit considered concerns the conic sections and the ellipse.

> "On the table. String models of the conic sections and ruled quadric surfaces (these were made in the school machine shop, but could easily be made from cereal boxes): drawing board and paper with two thumbtacks stuck in and a loose string tied around them ready to demonstrate the pin and string construction of an ellipse; an elliptic trammel or ellipsograph.
>
> Pictorial display. Poster A listed the members of the conic family, explained their family name by mention of their historical origin in their study by the Greeks of the right-angled cone, and showed by diagrams of elliptic and parabolic orbits one of their earliest applications, at the hands of Kepler and Newton."

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1. Eighteenth Yearbook, opp. cit. pp. 89-90.

> "Poster B showed two exact drafting-room constructions for ellipses, one using concentric circles, the other being a projective or so-called "parallelogram" construction".

Posters can be constructed around any mathematical topic which lends itself to any form of pictorial representation. While titles and written explanations are desirable and at times essential in a poster, written material should be kept to a minimum.

The use of posters is not restricted to bulletin board displays or exhibits. They may be used by the mathematics teacher to introduce a topic; to illustrate different aspects of the topic; and in some instances, to review the work covered in the consideration of the topic.

Bulletin Boards.

The bulletin board provides a variety of means of stimulating and directing the learning process of students. One feature which has its advantages as well as its disadvantages is its relative permanence and the fact that students are exposed for considerable time to the sensory appeal of bulletin board material. This may be turned into a disadvantage.if the material becomes stale through prolonged exposure or if the bulletin board becomes unattractive through neglect.

To help prevent this, each class should have its own bulletin board, and should assume responsibility for its care. Committees of students could be selected either

weekly or monthly and charged with the care of the bulletin board under the teacher's direction. Thus there would be given to all pupils an added sense of accomplishment, and a feeling that he or she is contributing actively to the educative process.

Too often is it the case that the bulletin board is so crowded with outdated notices, assignments, and the like that an observer would never guess that this were a Mathematics classroom, or one in which English or History, or Science were taught. A safe and a guiding rule to follow should be that the material displayed should be of immediate and direct applicability to the classwork under consideration. The material displayed should be significant educationally to the activities upon which the class is concentrating. The wise use of bulletin board materials also embodies the principles of variety and continuity as well as that of applicability.

The opportunity to provide a degree of artistic value in the arrangement of display materials should not be neglected. The fact that the bulletin board display is among other things, pleasing to the eye, adds much to its educational vlaue. The principles of balance proportion, color, line, and rhythm could be used to great advantage in arranging a display.

Care should be taken that the bulletin board does not

become sloppy and unsightly through neglect. Torn and disfigured clippings, smudged drawings, and the like are extremely undesirable. Little attention is paid by the students to a sloppy bulletin board. On the other hand, one which is pleasing to the eye does much to attract pupils' attention and gives the bulletin board an opportunity to make its contribution to the educational process.

An attractive and educational bulletin board display might be arranged in connection with statistics by posting some of the better graphs, (pie graphs, bar graphs, line graphs, and pictorial graphs) so common in current newspapers and magazines.

In connection with applications of mathematics to architecture, an excellent display might be arranged around pictures of modern skyscrapers, cathedrals, mosques, bridges, dams, storage tanks, etc.. Additional worthwhile activity for pupils in this field would be to show in the display, by posters, diagrams, graphs, etc., with what mathematical topic or topics the different illustrations are connected since this relation may not be apparent to some pupils.

Pictures of modern calculating machines, clipped from the literature supplied in great abundance by commercial concerns, supplemented by pupil-made diagrams and posters describing the operation and place of different machines in business and industry might be used for a bulletin board

display in connection with work in statistics, or conceivably, a brief unit of work on modern calculating machines. Such work would not be out of place in a mathematics classroom. These few suggestions are offered in the hope that they may give some small inkling of the contribution which a mathematical bulletin board display may make to furthering educational growth.

<u>Wall</u> <u>charts</u>.

Wall charts may be of the commercial or home-made type. The latter type may be made from pictures, charts, graphs, or diagrams enlarged to the desired size. While wall charts in the field of mathematics are not nearly as numerous as in the field of science, several excellent commercial charts are available (Sources #5 and #8 in Appendix).

Although wall charts can be made by enlarging pictures, charts, posters, etc., it is desirable that the subject for a wall chart be a bit broader than the single simple topic usually covered by a picture, chart or poster. Suggested topics for home-made wall charts might be:

1. The relation among families of theorems in geometry.
2. Applications of geometric principles in architecture or industrial design.
3. The construction of maps. (Show the different methods of projecting a rounded surface onto a flat surface.)
4. The conic sections.

These are but a few of the topics around which

excellent wall charts could be constructed by pupils with different interests and abilities.

Stereographs

The importance of stereographs is being realized more and more in connection with the study of plane and solid geometry. It is one of the most effective aids for the visualization of third dimension relationships. An excellent series of stereographs on solid geometry have been developed by John T. Rule[1]. In Visualizing the Curriculum,[2] the author of this series is quoted at length on the subject of these stereographs. Much of what he says is pertinent in the present discussion of stereographs:

> "The stereographs all represent standard solid geometry proofs found in the usual sequences of teaching the subject. Consequently, the lesson which goes with them merely consists of the proof of the particular proposition shown in whatever manner the teacher cares to develop that proof.
>
> "In my own experience with using these stereographs I have found that the removal of the strain of visualization has enabled the pupils to concentrate on the geometric principles involved and consequently they have enabled him to improve greatly his geometric insight.
>
> "It has been my practice to draw the identical picture on the blackboard, then allow each member of the class to look at the stereograph. I then outline the proof verbally and informally. I find that a student looking at the stereograph carries the impression and the understanding of depth over into the pland drawing so that he immediately sees it solid and is able to concentrate on the proof."

1. John T. Rule, Stereographs for Geometry, Keystone View Company, Meadville, Pennsylvania.
2. Charles F. Hoban, Charles F. Hoban, Jr., and Samuel B. Zisman, Visualizing the Curriculum, The Dryden Press,Inc. New York, 1937, pp. 151-152.

"This carry-over from the solid stereograph to the identical plane drawing very rapidly develops the power of visualization and is to my mind the outstanding feature that gives the stereograph its superiority over other forms of models.

"Where possible I have made a point of giving the informal verbal proof while the student is looking at the stereograph. The point of importance is that these proofs have been boiled down to their minimum as they should be given within the time that the student finds it comfortable to hold the stereoscope to his eyes. Furthermore I find that this brevity adds to the clarity of the proof."

A conceivable drawback in the use of stereographs might be the fact that they are not adapted to group teaching. The cost of equipping a class with stereographs and stereoscopes in sufficient numbers would doubtless be prohibitive in the majority of schools. In spite of this, it is to be hoped that more and more mathematics teachers will investigate stereographs with an eye to adapting their use to their classes in plane, solid, and analytic geometry. The ease with which stereographs capture and hold pupils' attention could do much to make instruction in these fields more interesting and meaningful.

Laboratory Equipment

<u>Construction paper, cardboard, etc., and models.</u>

In order to prevent overlapping and repetition, models and the materials necessary for their construction will be treated under a single heading. Commercial models will also be considered under this one heading.

In many instances, pupils will gladly furnish the supplies and equipment needed for the construction of mathematical models and devices. The school need have only such supplies and equipment as are needed for demonstration purposes or to illustrate the construction of certain models which may be required of all pupils. For the most part, however, construction of models will be done outside of class hours, and by a limited number of pupils. Construction of models has particular appeal for those students who have always enjoyed tinkering with gadgets and devices, and constructing model airplanes, ships, etc..

The fields of plane and solid geometry contain numerous topical areas which can be made more readily understandable by pupils through the use of models. Problems on locus, problems dealing with the volume and surface area of regular solids as well as problems dealing with spherical angles are but a few areas in which models could be used to advantage.

Algebra need not be omitted where models are concerned. One of the most effective models the writer has seen is concerned with the expansion of a binomial. This model can

be used in illustrating the expansion of the expressions $(a+b)^2$ and $(a+b)^3$ as well as for explaining the finding of square roots.

The material to be used in the construction of a model is determined by the use which will be made of the model and the desired degree of permanency. Models made from cardboard and string or from balsa wood are effective but easily damaged. On the other hand, models made from celluloid, wood, metal, or wire are relatively durable and can be used year after year.

Materials, equipment, and accessories which can be used for the construction of models are suggested in the lists which follow and are adapted from an article by Joseph Hilsenrath entitled "Materials for Mathematical Models: Their Selection and Use"[1].

Materials

1. Toothpicks
2. Wooden applicators
3. Venetian blind slats
4. Steel strapping
5. Cardboard
6. Bus-bar wire
7. Thin florist's wire
8. Steel wire circles
9. Plywood
10. Sheet metal
11. Brass strips and bars
12. Modelmaker's lumber
13. Balsa wood
14. Glass
15. Soap

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1. Eighteenth Yearbook, opp. cit. pp. 228-229.

16. Clay
17. Colored string
18. Plaster of paris
19. Plastic sheets
20. Plastic tube, bar, and rod

Tools

A. Razor blade type knife
B. Paper punch
C. Straightedge
D. Pair of scissors
E. Small hammer
F. Stapler
G. Pair of side-cutting pliers
H. Scriber
I. Metal punch
J. Eyelet punch
K. Soldering iron
L. Metal shears
M. Coping saw
N. Crosscut saw
O. Hack saw
P. Glass cutter

Accessories

a. Paper fasteners
b. Eyelets
c. Staples
d. Paste, glue, airplane cement
e. Rubber bands
f. Elastic, round and flat
g. Wire brads
h. Suction cups
i. Solder and soldering paste
j. Tape, masking, drafting, binding, etc.
k. Violin D-string
l. Thumb tacks
m. Gummed letters and figures
n. Paints, assorted colors
o. Leather strips
p. Golf tees

Ingenuity of teacher and pupils places almost no limit on the number and uses of models in the mathematics classroom and laboratory.

Commercial models are available for the teaching of mathematics. However, they will not be discussed in the

present study because it is firmly believed by the writer that pupil-made models contribute much more to educative growth than do ready-made models. Pupils can not only learn a great deal of mathematics from the use of models, but also must make use of mathematical principles in their construction of these models.

<u>Calculating machines</u>.

Calculating machines of all types are in almost universal use in business and industry. In view of this, it seems desirable that all pupils regardless of the type of curriculum in which they are enrolled, be taught something of the nature and operation of such machines. The emphasis need not and should not be on proficiency in the operation of these devices. It would be enough if all pupils could be acquainted with the fact that certain calculating machines are in existence and in current use, and could be taught the rudiments of operating one or two of these machines. If for no other reason than demonstration purposes, it is desirable to have at least one calculating machine in the mathematics classroom or laboratory.

Some schools offer courses in the mathematics and applications of statistics at the senior year and junior college levels. In such courses, calculating machines could be effectively employed in simplifying the processes involved in computing the mean and standard deviation, among other things.

In connection with courses in algebra, the principles involved in the operation of certain calculating machines could be tied up with the structure of our decimal number system, and the break-down of numbers into units, tens, hundreds, etc.. Problems in interest and arithmetic progressions could be solved by using some of the recently developed calculating machines.

In appraising the suggestions offered for the use of calculating machines in connection with mathematics courses, it should be remembered that the primary goal in the classroom is neither proficiency in their use nor is it illustration of their use in solving algebraic or statistical problems. The goal toward which teacher and pupils should move is an awareness of the existence of different calculating machines and a knowledge of their operation which does not strive to approach proficiency.

Filing cabinets.

Filing cabinets are seldom to be found in mathematics classrooms or laboratories. Under an adequate program of multi-sensory activities however, filing cabinets will be almost indispensable. Steel filing cabinets are almost unsurpassed when it comes to the preservation and storage of pictorial materials such as diagrams, posters, graphs, cartoons, stereographs, and photographs.

In connection with this aid, the task of indexing and filing pictorial materials could be intrusted to several

interested pupils, preferably girls. The task of classifying pictorial materials is an important problem. Usually, it will be up to the teacher to decide whether pictorial materials will be filed according to types, ie. graphs, charts, diagrams, posters, etc., or under the different mathematical topics covered by the graph, diagram or poster. If the former procedure is followed, an attempt will have to be made to classify the different types of materials under mathematical topics. The suggestions offered as to classification in no way exclude careful consideration of valuable suggestions which pupils may have on the subject.

One danger lies in letting the files just grow and become cluttered with useless material. The teacher should weigh carefully the merits of different material before turning it over to be filed for future use. A periodic check of materials in the files could be conducted with the object of removing material which has outgrown its usefulness or can be replaced by more useful material.

Exhibit cases.

Exhibit cases may be used to store as well as to display the better models and devices constructed by pupils. Their use need not be limited to the mathematics exhibit which the class may be planning. Placed in the laboratory, the exhibit case with models and devices will contribute to the mathematical atmosphere which should pervade the laboratory.

Located in the corridor, it may help to give parents and other visitors an idea of what is being done in the mathematics classes. A glass case with lights advantageously placed will afford protection to the models and other materials displayed while making these materials easily accessible for public inspection.

Slide Rules

Demonstration type.

Large demonstration slide rules, like the 7 ft. Mannheim type which can be purchased for about $8.00 can be used to illustrate the construction as well as the use of slide rules. A discussion of their construction could be carried on to greatest advantage in connection with the work on logarithms. The use of the slide rule should be taken up at an early stage in their study of mathematics in order that pupils may benefit from the ability to use the slide rule for computations in mathematics as well as in chemistry, physics, and related subjects.

For individual use.

Inexpensive slide rules of sufficient accuracy for classroom use can be purchased for as low as $.25. The price is considerably lower if slide rules are purchased in bulk by schools or school systems. In our democratic school system, this valuable aid to learning should be supplied to all pupils in connection with their work in mathematics. It is one thing for the teacher to manipulate one in front of

a class and to explain its use and construction. It is another thing to supply pupils with slide rules of their own to manipulate, and to demonstrate their use and construction by means of the large rule previously mentioned.

It would be completely in line with the activity program of a mathematics laboratory to devote one class session to the construction of slide rules. This would not consume as much time as some teachers may suspect. It could be accomplished very easily by gluing strips of logarithmic paper to strips of cardboard. Pupils could not construct a slide rule and fail to grasp and understand more fully the logarithmic principle underlying its construction.

Book cases and shelves.

The presence of a library in the school is no reason for omitting book cases and shelves from the classroom. Furthermore, if book cases are present in a classroom, they should not be kept locked and the books therein allowed to gather dust. If the pupils are to be expected to do collateral and supplementary reading in mathematics, provision should be made to provide for the exhibition of the necessary books in the classroom where they are readily accessible to all pupils. The chance that these books will read are thus greater than if they lay on the shelves of the school library.

<u>Mathematics books other than the regular text</u>.

The reading which may be done by pupils in mathematics courses on a required or voluntary basis may be divided into collateral and supplementary reading.

Pupils encountering difficulties in the study of a given mathematical topic may engage in collateral reading by studying the given topic as presented in several different texts. Other pupils may desire to read these texts to satisfy their desire for additional information. For this reason, there should be in the mathematics classroom or laboratory several copies of different up-to-date texts covering the material of the course.

Supplementary reading may take the form of recreational reading, reading on the history and development of mathematics, reading on applications of mathematics, and any other readings not directly connected with the topic being studied. This type of reading is desirable in that it can make the study of mathematics more enjoyable and more meaningful for a greater number of pupils. For this reason, the mathematics classroom should include,in all of the areas mentioned in connection with supplementary reading, several appropriate books.

Projection Facilities

Electrical outlets.

Not much need be said with respect to electrical outlets other than to mention that mathematics classrooms and laboratories should make adequate provision for the use of projection equipment. Outlets should be provided not only for the use of projectors but also for the use of other equipment and machines, such as modern calculating machines run by electricity. Particularly in connection with projected aids is it desirable that films, film strips, and lantern slides be shown in the familiar environment and atmosphere of the mathematics classroom or laboratory.

Screens, stationary and portable.

Since the projection of films, film strips, and lantern slides should be centered in the classroom, it is desirable that classrooms be equipped with portable or wall type screens. In selecting a screen, the mathematics department must consider several factors. Wall type screens are not readily portable and could not easily be transported from one mathematics classroom to another. This implies that every classroom would have to be equipped with this type of screen. Portable screens, on the other hand, would require more time and attention to set up.

Since the material used in different types of screens affects the type and brilliancy of reflection, this factor must also be taken into consideration.

Darkening facilities.

The most desirable arrangement in connection with darkening facilities is to have all windows equipped with opaque shades. Compromise measures might be the use of home-made shades of inexpensive canvas material or opaque drapery material. The latter would not only serve to adequately darken the room for projection purposes, but would add to the attractiveness of the room at other times.

Projection Equipment

Motion picture, lantern slide, and opaque projectors.

The projection equipment surveyed in the present study includes motion picture projectors, opaque projectors, and lantern slide projectors. A detailed account of the mechanics and operation of projectors, particularly motion picture projectors is not within the scope of the present study. Complete information on the different models and makes of projectors can be had by writing to any of the sources listed under the heading "Projection Equipment" in the Appendix. The present treatment will be limited to the discussion of the relative importance of the different types mentioned, and factors influencing their use in the classroom.

From the availability of material to be projected, it would appear that the opaque projector is the most useful of the three types mentioned. This is not to be interpreted as an attempt to minimize the value of lantern slide and motion picture projectors. However, the fact that an

opaque projector can project any flat or nearly flat material such as pictures, diagrams, graphs, cartoons, pupil papers etc. which are all readily available would tend to support the statement concerning their usefulness. The goals of quantity, quality, and variety has yet to be reached in the fields of lantern slides and motion pictures for the teaching of mathematics, a factor which prevents more widespread use of the related projection equipment. Lantern slides are available in the field of mathematics as are sound and silent motion pictures. If more extensive use were made of the materials now available, and their shortcomings made known to producers and distributors, the goal of quantity, quality, and variety in the field of mathematical films and lantern slides could be achieved much sooner than at present seems likely.

Since this projection equipment is most effective when used in the classroom or laboratory, their use in these places is dependent upon the existence of adequate darkening facilities, portable or permanent screens, and electrical outlets.

The portability of projection equipment is another factor which may affect the use of this equipment in the classroom situation. Early equipment was extremely bulky, but newer models are being produced with the object of greater portability.

Projected Aids

Sound and silent motion pictures.

When purchasing, renting, or making a motion picture, whether sound or silent, for the teaching of mathematics, the primary consideration should be, "Is motion an essential element of the material to be covered in the film?" As is implied by a leading authority in the field in an article in The Eighteenth Yearbook of the National Council of the Teachers of Mathematics[1], it would be a waste of time and good film to make or use a motion picture with pictures of pills, tin cans, and snowflakes to illustrate the many geometric shapes to be found in art, nature, and industry. In the words of this same authority[2]:

> "The secret is to use motion where it is implied in the mathematical argument, and where the expert and gifted mathematician supplies it with his intuition and imagination".

Teachers of mathematics would do well to keep these words in mind when contemplating the use or manufacture of films in their classrooms and laboratories.

The following suggestions adapted from an article in The Eighteenth Yearbook[1], represent general areas in which motion pictures could be used to advantage:

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1. Eighteenth Yearbook, opp. cit. p. 325.

2. Ibid. p. 326.

1. Geometric continuity
2. Limit theormes in geometry
3. Area of circles
4. The conic sections
5. Wave forms
6. Statistical concepts
7. Locus
8. Dimensions
9. Non-Euclidean geometry
10. Statistics and dynamics
11. Map projections
12. Symmetry (axes and points)

Sources of mathematical films are to be found in the Appendix. In addition,there is supplied through the courtesy of Mr. Henry W. Syer of Boston University's School of Education a fairly complete list of the films available for the teaching of mathematics with additional sources.

Film strips.

Film strips, just as motion pictures, are a comparatively recent development in the field of mathematics. Some of the more recent film strips for the teaching of mathematics, such as the Plane Geometry Series turned out by Curriculum Films[1] make effective use of color.

Inasmuch as film strips can be used in mathematical subject matter areas where motion is not an essential element, their range is broader than that of motion pictures. Suggested areas in which film strips can prove effective are the following:

1. Definitions of geometry's terminology and vocabulary.
2. Mathematical instruments (construction and use).
3. Optical illusions.

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1. See Source #31 in Appendix for address.

4. The fundamental processes
5. Signed numbers
6. Graphs in algebra and trigonometry
7. Exponents and logs
8. Foundations of geometry (axioms and postulates)
9. History of number
10. Geometric logic and deductive reasoning
11. Measurement-areas and perimeters
12. Logarithms

These are but a few of the areas in which film strips could be used to advantage. It will be seen that, on the whole, the areas suggested could be adequately covered by the film strips of from 25 to 50 frames. Titles and sources of film strips dealing with many of these areas are included in the Appendix. Under "Film Strips" in the Appendix are also given additional sources from which information concerning the more recent developments in film strips could be obtained.

Lantern slides.

In discussing lantern slides, no attempt will be made to discuss in detail the properties of the different types mentioned in the check list. The writer firmly believes that much more is to be gained from a discussion of lantern slides in general, and of their place in the mathematics classroom and laboratory.

One of the great advantages of lantern slides is that they can be used in almost all mathematical areas where motion is not an essential factor. Their value, however, varies in different areas as will be shown. Slides also have an advantage over films and film strips in that a set of slides may be arranged and projected in any order. In some cases, it may be desirable that certain slides in a series be omitted or that some be projected several times at different intervals during a given showing. This can be accomplished easily in the case of individual lantern slides. A film, on the other hand, must be run continuously from beginning to end, and the continuity of film strips cannot be changed.

In general, slides can be used to advantage in introducing a new subject, for drill, for review purposes, or even for presenting an entire unit of work.

Suggestions for general and specific topical areas in which lantern slides could be used to advantage might include:

1. Geometric forms in architecture and structural design.
   (A series of slides in this field might include pictures of modern skyscrapers, cathedrals and churches, bridges, dams, stadia, industrial plants, storage tanks, etc.)

2. History and development of numbers.
   (Illustrations of Sumarian, Egyptian, early Arabic, and Roman numerical symbols, together with illustrations of the different modifications of Arabic numerals down to the present form.)

3. Geometric forms in art and nature.

4. Graphs of trigonometric functions.

5. Statistical graphs.

6. Starred theorems in geometry.

7. Definitions in geometry.
   (Point, line, plane, circle, different types of triangles, regular polygons, etc.)

8. Percentage in general mathematics.

9. Reading gas and electric meters.

10. Arithmetic and geometric progressions in algebra.

11. Optical illusions.

12. Mathematical games and puzzles.

Experienced teachers of mathematics could doubtless add valuable suggestions to the list presented.

Suggestions for the evaluation of slides might prove valuable at this point particularly for those teachers who have never used this aid and may be contemplating the purchase or production of lantern slides.

A set of standards[1] representing the consensus of opinion of Directors of Visual Education follows:

1. Truth - Does the picture tell the truth; are the facts recorded accurately - free from distortion or illusion?

2. Photographic quality - Is the photography good; are the lines sharp, especially in the shadows; do the main facts stand out clearly in the midst of other details; is the material modern, not antiquated?

3. Relevancy - Does the picture pertain to and does it contribute meaningful content to the topic under discussion?

4. Relative Size of Items - Does the picture include items or elements of known size so that the observer may secure a correct idea of the unknown element?

5. Mechanical Qualities - Is the slide free from blemishes, smears, stains, scratches, blurs; is it substantially bound; does it contain a thumb mark?

A good slide serves to focus attention on one object at a time. It serves to clarify instruction, reduce abstraction, and present correct impressions. In some instances it frees the teacher from the task of illustrating on the blackboard and gives her added time to do a more effective teaching job. The production of lantern slides can provide meaningful and worthwhile activity for many pupils. These and other advantages can be discovered from experimenting with the construction and use of lantern slides.

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1. Visualizing the Curriculum opp. cit. p. 160.

Activities

School journeys.

Treatment of school journeys in the present study will be limited to suggestions for the use of this aid in connection with different mathematics courses. An admirable definition of school journeys is to be found in Visualizing the Curriculum:[1]

> "The school journey may be defined as an educational procedure in which pupils are conducted, for educational purposes, to places where the subject matter of instruction may be studied first hand in its functional situation".

Teachers of mathematics would do well to acquaint themselves with the opportunities for school journeys existing in a particular locality. In this connection, suggestions from pupils might prove valuable.

Suggested places for school journeys by mathematics classes might include banks, post offices, commercial houses, manufacturing plants, museums, bridges, and other places of mathematical interest which might occur to the resourceful and imaginative teacher.

In connection with a course in general mathematics, a visit to a bank might prove very valuable. Not only could the pupils observe a bank in operation, and see for themselves some of the practical uses of mathematics, but much information and printed material could be gathered for

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1. Visualizing the Curriculum opp. cit. p. 160.

practical problems in interest, savings bank life insurance, and financing.

A visit to a famous bridge or some other structure might prove interesting and educative to a class in plane or analytic geometry. The location of braces and struts might help to make problems on vectors and directed force more meaningful to pupils. The suspension of cables in suspension bridges might give added impetus and interest to problems on the catenary and related problems on maxima and minima.

These simple suggestions may serve to reveal some of the tremendous possibilities inherent in school journeys. Careful planning of the work which should precede and follow a school journey will help to make it truly a multi-sensory aid inasmuch as the pupils' energies and senses are coordinated in an "active" learning situation.

Construction of models.

As previously mentioned, the construction of models will be carried on by a limited number of pupils and seldom within regular school hours. The construction of models usually takes so much time that the value to be derived would not justify taking the time of an entire class. However, those pupils who will engage in this activity will take pleasure in making the other members of the class partners in the benefits which doubtless will result from the fruits of their labor. The benefits to be derived from working

with "concrete" objects and concrete illustrations cannot be minimized.

Simple pupil-made models of a cylinder and a cone of the same height will prove valuable in a class in plane and solid geometry. If the cone is filled with sand and the sand is then poured into the cylinder the pupils can see it proved before their very eyes that the volume of a cone is one-third that of a cylinder. In the same way, by using two hollow cubes with the side of one twice that of the other, changes in area and volume when the side of a cube is doubled can be concretely illustrated. A simple circular cardboard model which can be rotated about a pivot with a piece of weighted string attached to the rim can be used in trigonometry classes to illustrate concretely how the value of the different trigonometric functions changes with a change in the size of an angle.

It is easy to see by these suggestions for models, and by the many models which ingenious students will produce, how much can be accomplished in moving from the abstract to the concrete. The chance to make, manipulate, and experiment with models as part of mathematical proofs will make the work of teacher and pupils more interesting than could be hoped for under the traditional method of instruction. The pupils will be given the opportunity to discover mathematical principles for themselves, and once having done so, will understand these principles better.

<u>Planning and working on exhibits.</u>

An exhibit provides mathematics classes with an excellent medium for showing that mathematics is alive and interesting and not a mass of difficult-to-understand signs and symbols. Particularly will this be true if an attempt is made to dramatize and illustrate the many practical applications of mathematics in everyday life. An exhibit may be said to achieve its highest goal if it proves entertaining and educational to the pupils who prepare it as well as understandable and educational to those with meagre mathematical background who will view it.

The many and varied activities involved in the planning, preparing, and conducting of exhibits makes possible the enrollment of the energies of pupils possessing different abilities and interests. Hence, one great educational objective partially achieved will be provision for individual differences. In fact, not only through this aid, but also through the use of most of the aids considered in the present study can provision be made for differences in abilities and interests.

In planning the exhibit, much thought will be given by the pupils to the general theme of the exhibit as well as to the objectives to be achieved by different parts of the exhibits. Once the objectives have been set up, thought and action will be directed towards ways and means of achieving

these objectives. The site of the exhibit will have to be chosen and equipment such as tables, showcases, and apparatus will have to be assembled and arranged. Models will doubtless be required for different parts of the exhibit. These, together with posters to explain different parts of the exhibit will be made by pupils. Pupils will be posted by different displays to explain the different mathematical principles involved to visitors. The exhibit, to be successful, will have to be "sold" to the school and the community. A well planned exhibit will not want for "salesmen" among the members of the class.

An excellent illustration from an actual exhibit reported in <u>The Eighteenth Yearbook</u>[1] might prove useful at this point. The part of the exhibit to be quoted dealt with the parabola, catenary, maxima, and minima:

> "<u>On the table</u>. Stoppered glass tube partly filled with a colored liquid and mounted so that it could be rotated about its vertical axis, thus showing the parabolic shape of the surface of a rotated fluid and demonstrating a device which, when geared to the drivers and properly calibrated, served as a speedometer on some early trains, the speed being determined by the height to which the oil would rise.
>
> "Also on the table were two pairs of ringstands of the same height, side by side, from which were suspended two cables of the same length, one loaded horizontally to show a parabolic shape, the other a free-hanging catenoid cable. (To have this set up as accurately as possible, the "cables" should be flexible and the load suspended from the one should be large in relation to the weight of the cable.)

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1. opp. cit. pp. 91-93

"A soap solution and wire forms for demonstrating minimal surfaces, and two tin cans, found in a grocery store, which had the same volume but different shapes and surface areas completed the array of apparatus."

"Pictorial display. Poster A showed a picture of the Detroit-Windsor Ambassador Bridge and of a high tension line. Poster B was a diagram for and solution of the problem, of determining the dimensions necessary to give the beam of maximum strenght which can be cut from a given log.

"The student. The student began his discussion by explaining wherein the rotating fluid demonstrated another occurrence of the curve which the spectator had met in the previous section (of the exhibit). He then called attention to the parabolic cables of the Detroit-Windsor Ambassador Bridge but emphasized that free-hanging cables, such as high tension wires, are not parabolic, but catenoidal. The catenary, he continued, is a curve with many interesting properties and uses in its own right; for instance, it is the sail curve, as shown by fastening a handkerchief to parallel uprights, and allowing a fan to blow on it; it likewise provides the basis for computations made by surveyors in correcting for the sag in steel tapes and chains; it finds a place in architecture; and, as demonstrated by the soap solution and wire forms, it provides the curve which, when rotated, traces out the surface of least area which can be used to join parallel circular rings. The problem of "least" and "largest" incidentally, the student explained, occurs in many places in mathematics and its applications, such as in determining the dimensions for the strongest beam that can be cut from a given log or in determining the dimensions for a tin can which will give the minimum area for a fixed volume."

References for further readings on exhibits can be found in the supplementary bibliography in the Appendix. The possibilities for educational growth through exhibits can best be appreciated by planning and conducting such an exhibit as has been quoted in the present section.

<u>Mathematical games and contests</u>.

The competitive spirit present in so many children can be turned to advantage in the mathematics classroom situation, and a child's natural desire to compete and excel used to further his educational growth. Some might argue against this type of activity on the grounds that it is an artificial way of stimulating interest. However, it should be stated that this type of activity should not be allowed to evolve into a three-ring circus, nor should it be used too frequently. If properly planned and conducted, with an attempt made to have pupils see that this type of activity is an educational process and not mere entertainment, much can be done to enliven instruction and stimulate thinking along mathematical lines.

Several of the references in the Appendix may offer suggestions for games and contests adaptable to many mathematics classrooms.

Games and contests may take the form of quiz programs with the class evenly divided into groups. One of the by-products of such a division will be a surprising amount of team spirit. It is obvious that questions must be brief, easily understood, and capable of being answered quickly without the need for excessive computations. Tricky questions should be kept to a minimum. The activity should be a teaching-learning situation, and not an attempt to fool or show up

the pupils. The adaptation of this type of activity to different mathematical subjects requires, (among other things) a thorough knowledge of the subject matter and the ability to keep the class under control once pupil enthusiasm is aroused. The latter may present a much more difficult problem than appears on the surface.

Another type of contest may concern models, posters, pictures, and other objects made or collected by pupils. A statement to the effect that the best of this type of material will be regularly posted, or displayed in an exhibit under the pupil's name will do much to arouse competitive spirit and stimulate activity along this line.

In connection with games and contests, the teacher should never lose sight of the many valuable suggestions which pupils can make along these lines. Pupils might be encouraged to turn in suggestions for games and contests together with clear written instructions for conducting the game or contest. An analysis of these suggestions will permit the teacher to schedule those which appeal to the greatest number. If this is done, the task of arousing interest in what may be an artificial teacher-made game or contest will be eliminated. All normal children like to play and compete. Time spent on devising ways and means of taking advantage of this natural tendency to further the study of mathematics would be time well spent.

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Mathematical plays and dramas.

Plays and dramas, just as the exhibits previously mentioned, provide a means for dramatizing the role which mathematics plays in our everyday life. However, the time which may be consumed in the production and presentation of a mathematical play may outweigh the benefits to be derived. Williams[1] offers two remedies for this:

> "One is to be sure that dramatization is not used too frequently and that when it is used, it warrants the time expended. The other remedy is to keep properties and costumes to a minimum, even to the point of no properties and no unusual costumes............something should be left to the imagination and resourcefulness of the spectators".

A mathematical play or drama might well be presented in connection with the activity program of a mathematics club under the supervision of a teacher sponsor. The work of selecting a theme, writing the script, preparing costumes and scenery, if any, casting, etc. should be left up to a committee of pupils if compatible with an effective production. It can be seen that the activities involved in the production of a play allow for many different interests and abilities.

A good general these for a play might be, "Life Without Mathematics". Such a theme would lend itself easily to dramatization. It would also require a good deal of study on the part of students during which the practical applications of mathematics down through the ages would be

1. L.A. Williams, op. cit. p. 442.

investigated. The wealth of material uncovered would doubtless necessitate delimitation of the original theme. Even after this delimitation, the pupils may find that they have too much material. The process of selecting from this material may require a bit of critical evaluation on the part of pupils. The possibilities for furthering educational growth in mathematics through plays and dramas are great.

Plays based on the historical background and development of mathematics could do much to develop an appreciation of this study. Conceivably, a play designed to bring out the lighter side of mathematics, its "magic" and entertainment value might be planned to contribute to an appreciation and greater understanding of number and its properties. These few suggestions only begin to enumerate the possible mathematical areas about which plays could be prepared, and the values to be derived from a well planned and well executed performance.

Mathematical recreations and amusements.

The inclusion of recreation and amusements in a mathematics course is in agreement with the psychological laws of learning, and will do much to make the study of mathematics more interesting and enjoyable to pupils. A great deal of this work might be done under the auspices of the mathematics club.

One difficulty in attempting to isolate recreations and

and amusements in mathematics lies in the overlapping and interrelation of this aid with mathematics clubs, games and contests, and plays and dramas. It is easy to see how recreations and amusements could fall into any or all of the three other spheres of activity mentioned. This integration is desirable. For the purpose of the present study, however, an attempt will be made to isolate recreations and amusements and to offer a few suggestions for activities which may fall into this classification.

In connection with geometry, for example, let us consider something which appeals to many, old and young alike, a crossword puzzle. Would it be possible to construct such a puzzle based on definitions and vocabulary in geometry? Would solving such a puzzle lend interest to the study of definitions and vocabulary in geometry? Would this make more enjoyable the achievement of certain objectives in connection with a geometry course? The present discussion, however, must not be interpreted to mean that frequent use of such an aid should be made or that instruction in geometry should be turned into a puzzle or a game. Common sense will guide a teacher in the use of such an aid.

Much can be done in an algebra class with recreational problems dealing with the determination of a number selected by someone or the prediction of the result of certain operations. In connection with these, pupils may be asked

to explain the solutions algebraically. The crossword puzzles previously mentioned could be adapted to algebra courses if numbers were substituted for letters. It is conceivable that pupils themselves may develop problems along these lines.

Arithmetical fallacies and paradoxical problems may prove educational as well as entertaining. The rules for the construction of different types of magic squares may amuse pupils. Supplying the mathematical foundation for these rules may give them food for much thought. Problems with cards may be adapted to classes in albegra and geometry. Up to a certain point, work on the three famous problems of antiquity may be considered recreational. Problems in probability, combinations, and permutations can also be developed along recreational lines.

Much has been written in the field of recreations in mathematics. Several of the articles listed in the Appendix deal with actual classroom use of recreational problems. In addition to these, the following list of books is presented in the hope that teachers may find in them suggestions for adding zest and new interest to their mathematics courses:

1. Abraham, R.M.: "Winter Nights Entertainment" (New York: E.P.Dutton & Company, Inc.)

2. Andrews, W.S.: "Magic Squares and Cubes" (La Salle, Illinois: The Open Court Publishing Company)

3. Ball, W.W.: "Mathematical Recreations and Essays" (New York: The Macmillan Company)

4. Collins, A. Frederick: "Fun With Figures" (New York: D. Appleton-Century Company, Inc.)

5. De Morgan, Augusta: "A Budget of Paradoxes" (La Salle, Illinois: The Open Court Publishing Company)

6. Dudeney, Henry E.: "Amusements in Mathematics" (New York: Thomas Nelson & Sons)

7. Heath, Royal Vale: "Mathemagic" (New York: Simon & Schuster, Inc.)

8. Hogben, Launcelot: "Mathematics for the Million" (New York: W.W.Norton & Company, Inc.)

9. Jones, S.I.: "Mathematical Clubs and Recreations" (Nashville, Tennessee: published by the author)

10. Kasner, Edward and James Newman: "Mathematics and the Imagination" (New York: Simon & Schuster,Inc.)

11. Licks, H.E.: "Recreations in Mathematics" (New York: D. Van Nostrand Company, Inc.)

12. Rupert, William W.: "Famous Geometrical Theorems and Problems" Parts I-IV (Boston: D.C.Heath & Company)

13. Sloane, T.O.: "Rapid Arithmetic" (New York: D. Van Nostrand Company, Inc.)

14. Thompson, J.E. and H.E. Licks: "Speed and Fun With Figures" (New York: D.Van Nostrand Company, Inc.)

15. Smith, D.E.: "Number Stories of Long Ago" (Boston: Ginn and Company)

16. Woodring, Maxie, and Vera Sanford: "Enriched Teaching of Mathematics in the Junior and Senior High School" (New York: Bureau of Publications, Teachers College, Columbia University)

Mathematical Clubs.

Mathematics clubs are almost unsurpassed as a medium for arousing and stimulating interest in the mathematical studies. This may be due in part to the type of pupils who make up the membership of such a club, and to the freedom with which questions can be raised and discussed and opinions expressed on subjects brought up by the pupils themselves. A teacher's sole duty in connection with club activities should be to act as a guide and counsellor. She should be more a spectator than an active participant. If pupils are ever to be able to formulate plans and to carry out these plans of their own, the chance to do so in connection with the running of a mathematics club should not be denied them. The mathematics club will afford them an opportunity to grow in self direction as well as in their knowledge of mathematics.

Topics to be included in a mathematics club program may be drawn from the history and development of present-day mathematics, biographical sketches of men who played a prominent part in this development, recreations and amusements in mathematics, applications of mathematics to every day life and to other fields of study, and aspects of mathematics seldom covered in the classroom. Discussion of these topics will benefit primarily the club members. As a result of these discussions, however, the mathematics

club may decide to present an exhibit or a play, or plan a mathematical program for the school assembly to benefit other students of mathematics and the entire student body.

Teachers would do well to investigate the possibilities of starting mathematics clubs for the different grade levels of mathematics taught in their school. It is desirable that a club limit its membership to a single grade level, this membership being on a purely voluntary basis. A homogenous membership will go far towards removing obstacles in the path of planning an activity program in which all can participate and from which all can benefit. Suggested readings on the subject of mathematics clubs and club programs can be found in the Appendix.

CHAPTER III

MULTI-SENSORY AIDS ACTUALLY USED IN MATHEMATICS CLASSROOMS AND LABORATORIES

Method of Procedure and Sources of Data

Distribution of Check Lists.

The check list to be found in the Appendix was mailed to every public high school in the State of Massachusetts. The number of these check lists came to 258. In addition, 12 copies were mailed to public high schools in Maine, New Hampshire, Vermont, Rhode Island, Connecticut, and Pennsylvania upon request by individuals interested in the study. These 12, however, do not comprise a sufficiently large or representative sample to make any significant contribution and shall be disregarded in reporting the results of the study.

Of the 258 mailed to secondary schools in Massachusetts, 150 or 59.7 per cent had been returned at the time tabulation was begun. To date, 27 additional check lists have been returned, bringing the total return to 177 or 68.6 per cent. This study, however, will be based on the original 150 returns, and all tables, graphs, and charts will be based on a return representing 59.7 per cent of the schools contacted.

Geographical Distribution of Returns on Check List.

The outline map on the following page shows the geographical distribution of the schools cooperating in this

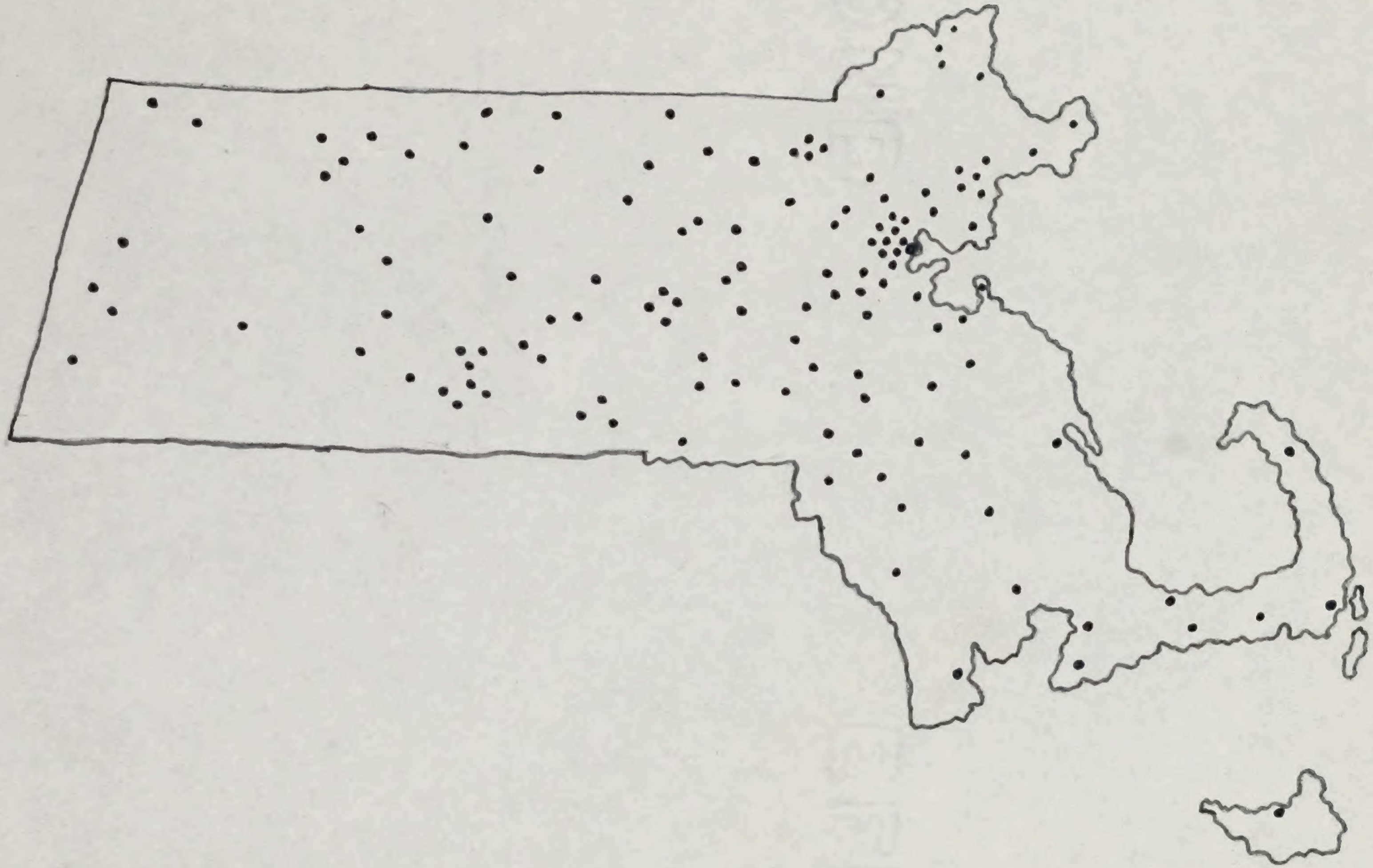

Fig 1. Geographical Distribution of Check List Returns

study. It can be seen that in spite of a significant clustering of dots about such large cities as Boston, Worcester, and Springfield, the overall distribution appears to be fairly representative. It should also be mentioned that the cluster of dots about Boston represents the school systems of Newton, Watertown, Malden, Somerville, Medford, Everett and other cities and towns in the environs of Boston. There were 9 returns from the secondary schools of Boston proper, and these are represented by the single large dot to the right of the cluster.

Validity of the Present Study.

In answering the question, "Is the present study valid?", we are in reality asking, "Will the present study reveal the status of multi-sensory aids in the mathematics classrooms and laboratories of Massachusetts secondary schools? The check list was constructed with an eye to determining the degree to which classrooms and laboratories of the mathematics department were equipped with these aids, and the use that was made of some of these aids. This, it is felt, adequately covers the question of "status".

In a previous chapter, the term "multi-sensory aids" has been defined. In the chapter on illustrations of uses of different aids, it has been shown that the aids listed in the check list conform to the definition given. Hence, the study will reveal the status of aids which are not purely auditory or visual, but multi-sensory in nature.

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One might question the validity of the present study by saying that many of the aids listed in the check list can be and have been used in departments other than mathematics. The obvious truth of such a contention could not be denied. By the same token, neither can it be denied that the aids listed can and have been used in mathematics classrooms and laboratories. The fact that heads of mathematics departments were requested to complete the check lists gives a reasonable assurance of a valid report on the status of these aids in these departments.

In concluding the discussion on validity, the question might be raised as to whether the sum total of the schools reporting is representative of Massachusetts secondary schools as a whole. In reply to this question, it can be mentioned that the figures reported are based on 150 schools are 59.7 per cent of the schools contacted. The outline map representing the geographical distribution of check list returns previously referred to in this chapter illustrates graphically a reasonably random sampling or a representative return.

By defining "status" and "multi-sensory aids" as interpreted in the present study and mentioning that the check list was constructed with these definitions in mind, together with a discussion on the representativeness of the return, the writer has attempted to show that the study

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will measure what it purports to measure and is hence valid.

Preparation of tables and graphs

In reporting the results of thepresent study in tabular and graphical form, percentages are given to the nearest tenth of a per cent. Multi-sensory aids falling into the same general classification will be treated as a group.

Blackboards and blackboard equipment.

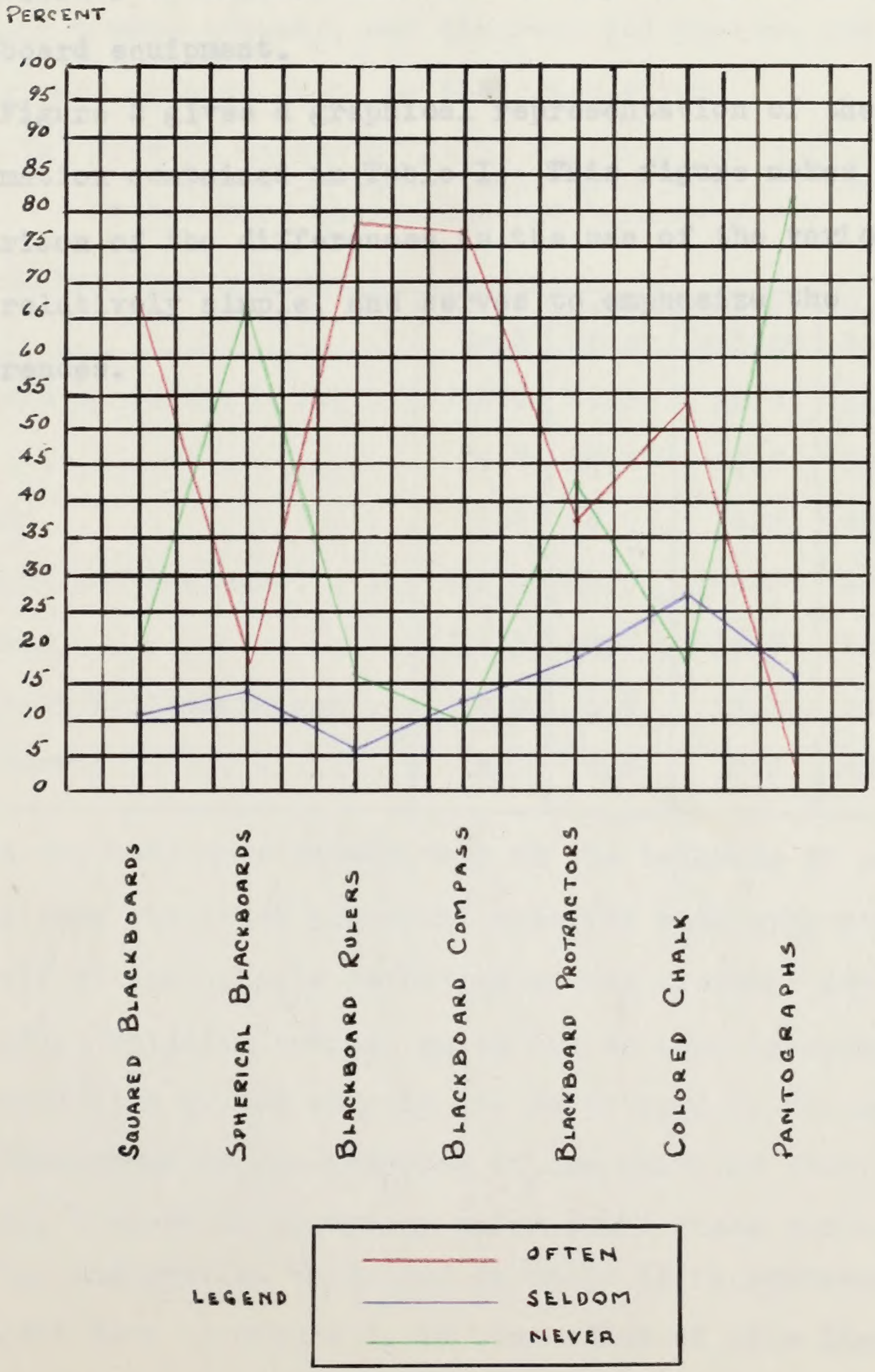
Heads of Mathematics Departments were requested to report on the status of these aids by indicating whether the aids were on hand, and the degree to which the aids were used. Table I is a summary of the results.

Table I Present Status of blackboards and blackboard equipment and degree to which they are used in mathematics classrooms and laboratories

	On Hand	Used Often	Used Seldom	Never Used
Squared Blackboards. . . .	80.7	68.7	10.7	20.7
Spherical Blackboards. . .	32.7	18.7	14.0	67.3
Blackboard Rulers.	85.3	78.0	6.0	16.0
Blackboard Compass	92.7	77.3	12.7	10.0
Blackboard Protractors . .	58.0	37.3	19.3	43.3
Colored Chalk.	84.0	54.0	27.3	18.7
Pantographs.	18.0	2.7	15.3	82.0

It will be seen from this table that the majority of schools are well equipped with the exception of spherical blackboards and pantographs and make extensive use of the aids listed. It will be noticed that approximately 2/3 and 4/5

Figure 2 — Degree to Which Blackboards and Blackboard Equipment Are Used in Mathematics Classrooms and Laboratories.



of all schools never use spherical blackboards and pantographs respectively. On the whole, it would appear that the majority of schools make reasonably extensive use of blackboard equipment.

Figure 2 gives a graphical representation of the information contained in Table I. This figure makes comparison of the differences in the use of the various aids relatively simple, and serves to emphasize the differences.

Still pictorial materials.

A survey of the status of still pictorial materials, the results of which are reported in Table II, reveals that bulletin boards, graphs, and diagrams and posters are the more common aids on hand, in the order given

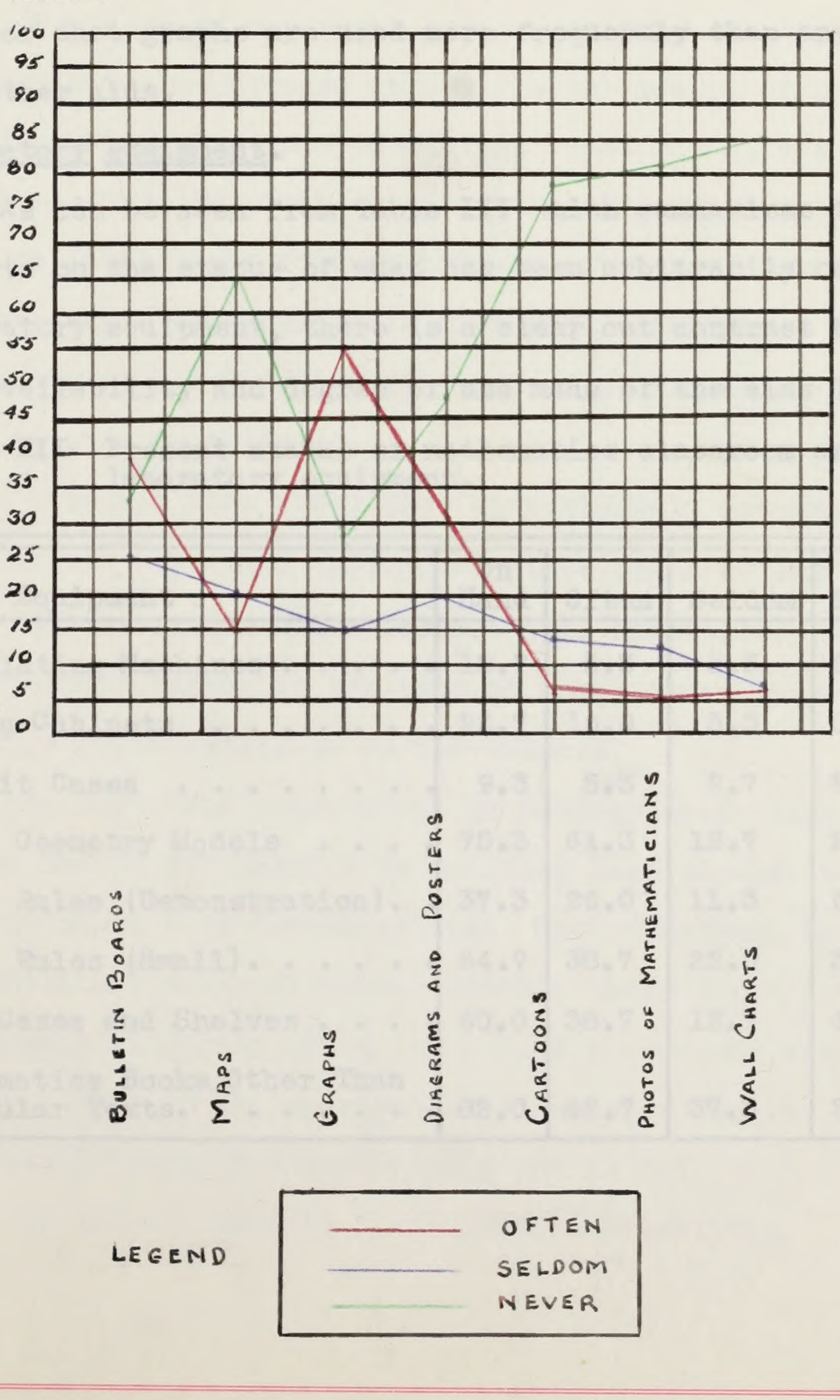
Table II Present Status of Still Pictorial Materials and the degree to which these materials are used in mathematics classrooms and laboratories.

	On Hand	Used Often	Used Seldom	Never Used
Bulletin Boards.	82.0	39.3	26.7	34.0
Maps	36.7	15.3	20.0	64.7
Graphs	71.3	56.0	15.3	28.7
Diagrams and Posters	52.0	32.0	20.0	48.0
Cartoons	21.3	6.7	14.7	78.6
Photos of Mathematicians . .	20.0	6.0	13.3	80.7
Wall Charts	16.0	7.3	7.3	85.4

Graphs are used more extensively in the teaching of mathematics than any other pictorial material with approximately one-half of the schools reporting making frequent use of this aid. Bulletin boards, which are on hand in approximately four-fifths of the schools are never used in the mathematics classrooms or laboratories of one-third of these schools, a waste of equipment which could stand correction.

The information contained in Table II is represented in graphical form in Figure 3. An inspection of this figure

FIGURE 3 — DEGREE TO WHICH STILL PICTORIAL MATERIALS ARE USED IN MATHEMATICS CLASSROOMS AND LABORATORIES.



reveals at a glance the difference in the use of the different aids listed. One need not pore over a maze of figures to see that, in most schools, wall charts are never used or that graphs are used more frequently than any of the other aids.

Laboratory equipment.

As can be seen from Table III which summarizes teacher reports on the status of what has been arbitrarily called laboratory equipment, there is a clear cut contrast between the availability and degree of use made of the aids listed.

Table III Present status of mathematics classroom and laboratory equipment.

Equipment	On Hand	Often	Seldom	Never
Calculating Machines	12.7	5.3	5.3	89.3
Filing Cabinets	22.7	16.0	3.3	80.7
Exhibit Cases	9.3	5.3	2.7	92.0
Solid Geometry Models	75.3	61.3	12.7	26.0
Slide Rules (Demonstration). .	37.3	26.0	11.3	62.7
Slide Rules (Small).	64.7	38.7	22.0	39.3
Book Cases and Shelves	60.0	38.7	12.7	48.7
Mathematics Books Other Than Regular Texts.	82.0	42.7	37.3	20.0

Calculating machines, filing cabinets, and exhibit cases are not available in approximately four-fifths of the schools reporting.

Such lack of equipment is to be deplored not only in itself, but also because it affects the use of other aids. The presence or absence of exhibit cases may affect work on models and the planning and conducting of exhibits. The availability of filing cabinets is closely related to the use and storage of still pictorial materials. These examples may serve to illustrate the interrelation among certain aids to learning.

Table III also shows that approximately one-half of the schools reporting individual slide rules on hand have large demonstration slide rules. This discrepancy may be an important factor in the type of instruction in the use of slide rules available in the schools.

The fact that four-fifths of Massachusetts high schools report having mathematics books other than regular texts on hand might bear investigation. Unfortunately, the present study does not reveal whether these books are in the school libraries or in the actual classrooms and laboratories. This is an important consideration for the use that pupils make of these books might be said to be directly proportional to their ready availability.

Projection facilities.

Heads of mathematics departments were asked to report on how well equipped their mathematics classrooms and laboratories were with respect to electrical outlets, screens, and darkening facilities. Table IV is a summary of the reports in this area.

Table IV Percentage of mathematics classrooms and laboratories equipped with electrical outlets, screens, and opaque shades or other darkening facilities.

	Yes	No
Electrical Outlets.	63.3	36.7
Screens	42.7	57.3
Shades and Other Darkening Facilities .	50.7	49.3

Approximately half the schools reported that screens and darkening facilities were not available. Some of these schools reported having a "visual aids" room which all departments used for the projection of films, lantern slides, film strips, etc.. Others reported that the auditorium is used for this purpose. However, since optimum benefits are derived only when the projection of films, film strips, lantern slides and other aids can be carried out in the familiar environment of the mathematics classroom or laboratory, it is to be hoped that existing deficiencies can be corrected by the purchase of opaque shades and stationary or portable screens.

The fact that slightly more than one-third of the schools reported a lack of adequate electrical outlets poses a rather difficult problem, since the situation reported is not one which can be easily corrected. Schools built prior to the introduction and use of projection equipment in the classrooms did not, understandably, provide electrical outlets for the use of such equipment. While this lack of outlets can be circumvented by improvising with the electrical facilities on hand, it is not desirable or recommended. Since films and other projected aids are here to stay and will be used more and more extensively, it is to be hoped that those concerned with the construction of new educational buildings will make adequate provision for the use of projected aids in the individual classrooms.

Projection equipment.

In investigating the status of certain specified projection equipment, information was requested under five different headings: A, B, C, D, and E. The meaning of these symbols follows Table V which reports on the status of projection equipment. In subsequent tables, the meaning of these symbols will remain the same unless otherwise stated.

Table V Present status of projection equipment.

	A	B	C	D	E
Opaque Projector	2.7	25.3	5.7	66.7	14.0
Lantern Slide Projector.	3.3	44.7	6.7	45.3	12.0
Sound Film Projector	10.0	61.3	7.3	21.3	5.3

A: Available in room at all times.
B: Available in school upon request.
C: Available in school system upon requisition.
D: Not available.
E: Should like to have the aid available.

The majority of schools appear well equipped as concerns sound filmpprojectors with 71.3 per cent or 107 schools out of 150 schools reporting this aid available either in the classroom or in the school. An interesting question, not answered by the present study, might concern the availability for use by the mathematics department of the school projector usually to be found in the auditorium. Such a projector, if not available for classroom use, contributes little to a school's educational program. In schools where money for such equipment is scarce, the question of greater utilization of existing equipment is an important one.

Table V reveals that 28 per cent of the school contacted have opaque projectors and 48 per cent have lantern slide projectors. Again in this situation it is hoped, but not ascertained in the present study, that these aids are readily available for use by the mathematics department,

in the mathematics classroom if possible.

As shown in Table V, two-thirds of the schools reporting do not have opaque projectors available, and approximately one-half have no lantern slide projectors. An interesting revelation is the fact that only 14 per cent and 12 per cent of the schools would like to have available opaque projectors and lantern slide projectors respectively. The present study does not attempt to explain this deplorable situation. On the whole, it would appear that schools stand in need of more projection equipment.

Lantern slides.

Although differences in the status of different types of slides are slight, plain glass slides are available in more schools than is any other type. However, the general impression received from an inspection of Table VI is that very few schools have any lantern slides at all, and furthermore, very few schools desire this type of aid in their mathematics department.

Table VI Status of different types of lantern slides.

Type of Slide	A	B	C	D	E
Plain Glass	.7	16.0	5.3	78.0	8.7
Ground Glass	.7	8.7	4.7	86.0	8.7
Cellophane	0	10.0	3.3	86.7	8.0
Silhouette	0	5.3	2.0	92.7	7.3
Photographic	.7	10.7	4.0	84.7	9.3
Kodachrome	0	10.7	3.3	86.0	10.0

A: Available in room at all times.
B: Available in school upon request.
C: Available in school system upon request
D: Not available.
E: Should like to have the aid available.

Of the 128 odd schools in which lantern slides are not available, only approximately 13 schools would like the aid available. Inasmuch as the present study is concerned mainly with the status of different multi-sensory, no attempt will be made to explain why the mathematics departments are apparently so resistant to the inroads of different aids.

Table VII Status of sound films listed in the check list.

Title	A	B	C	D	E
Origin of Mathematics.	0	4.0	8,7	87.3	34.7
Geometry in Action	0	4.7	8.0	87.3	36.7
Rectilinear Coordinates.	0	4.7	8.7	86.7	32.7
The Slide Rule	0	4.0	8.7	87.3	33.3
Precisely So	0	3.3	8.0	88.7	32.0
Locus.	0	4.0	8.7	87.3	36.0
Lines and Angles	0	4.0	9.3	86.7	34.0
Angles	0	3.3	8.0	88.7	34.0
The Circle	0	2.7	8.7	88.7	34.7
Chords and Tangents of Circles . . .	0	2.7	8.0	89.3	35.3
Angles and Areas in Circles.	0	2.7	8.0	89.3	34.7
Vernier Scale.	0	2.7	8.0	89.3	32.7
Long Division.	0	2.7	8.0	89.3	28.7
Property Taxation.	0	4.0	8.0	88.0	27.3
What is Four?	0	2.7	7.3	90.0	26.0
Average	0	3.5	8.4	88.2	32.8

A: Available in room at all times.
B: Available in school upon request.
C: Available in school system upon requisition.
D: Not available.
E: Should like to have the aid available.

<u>Sound films listed in check list</u>.

The sound films included in the check list were selected to include many of the older mathematics films available as well as some of the more recent ones. The uniformity of the results under certain headings as revealed by Table VII can be interpreted in several different ways. It may be that this section of the check list was not carefully filled out by those reporting with little attention paid to the individual films. The other conclusion may be that, in a given school, all films are available upon request or none at all.

On the whole, Table VII reveals several striking facts:

1. Not a single film listed is available in a mathematics classroom at all times.
2. Approximately 5 schools, out of 150 reporting, have films available in the school upon request. (This can be interpreted to mean that the films are actually in the school or can be sent for.)
3. In approximately 133 schools, sound films are not available.
4. Approximately 49 out of 150 schools would like to have sound films available.

These facts can lead only to the conclusion that very few, if any mathematics films, are used in the majority of Massachusetts secondary schools.

Silent films listed in check list.

A glance at Table VIII will show that the status of silent mathematics films roughly parallels that for sound films.

Table VIII Status of silent films listed in check list.

Title of Film	A	B	C	D	E
Frequency Curves.	0	3.3	6.7	90.0	21.3
The Isograph.	0	2.0	6.7	91.3	20.7
Our Children's Money	0	2.0	7.3	90.7	21.3
Einstein's Theory of Relativity . . .	0	2.7	8.7	88.7	24.0
Average	0	2.5	7.4	90.2	21.8

Very few silent mathematics films are available, with 90.2 per cent of the schools reporting them as not available. Stated a bit differently, 9 out of every 10 schools have no silent films available for the teaching of mathematics,

and only 1 out of every 5 schools would like to have this aid available. It would appear that mathematics films in general have yet to make any significant impression in present methods of teaching mathematics.

Film strips listed in check list.

The status of film strips parallels that of sound films even more closely than does that of silent films. This can be seen by comparing Table IX with Table VII.

Table IX Status of film strips listed in check list.

Title of Film Strip	A	B	C	D	E
The Origin of Algebra	.7	2.7	6.7	90.7	32.0
Basic Definition of Algebra	.0	2.0	7.3	90.7	29.3
Introduction to Plane Geometry. . .	.0	2.7	7.3	90.0	34.0
The Circle	.0	2.7	7.3	90.0	29.3
Angular Measurement	.7	3.3	7.3	88.7	31.3
Constructions	.7	2.7	7.3	89.3	31.3
Positive and Negative Numbers . . .	.7	3.3	7.3	88.7	32.0
Ratio and Proportion.	.7	3.3	7.3	88.7	32.0
Exponents and Logarithms	.7	2.7	7.3	88.7	31.3
Rectilinear Figures	.7	2.7	7.3	89.3	32.0
Math Instruments Series (5 strips).	.0	2.0	6.7	91.3	31.3
Plane Geometry (color-16 strips). .	.0	2.0	6.7	91.3	32.7
Average	.4	2.7	7.2	89.8	31.5

It is apparent that film strips are no more available than are sound and silent films. Approximately one school in ten has this aid available or can request it. Of those schools which do not have the aid available, one school out of every three would like to have it available. As previously mentioned, no attempt will be made to explain why comparatively few of the schools which do not have the aid available desire the aid.

Table X Present relative status of lantern slides, sound films, silent films, and film strips.

	A	B	C	D	E
Lantern Slides.	.7	20.7	7.3	71.3	13.3
Sound Films	.0	11.3	9.3	79.3	27.3
Silent Films.	.0	5.3	8.0	86.7	20.0
Film Strips	.7	6.7	6.7	86.0	28.7

Lantern slides, sound films, silent films, and film strips.

Table X might be considered a check on the status of sound films, silent films, and film strips in general as compared with that of the selected films and film strips listed in the check list. If the results reported in this table differed to any significant degree from those reported for the selected films and film strips, the selection of films and film strips might be questioned. However, inasmuch as there is little difference, the comments made can stand unchanged.

Table XI Status of stereographs.

	A	B	C	D	E
Stereographs.	3.7	2.0	2.0	92.7	12.0

Stereographs.

Table XI reveals the status, in mathematics classrooms and laboratories, of an aid which was perfected about the

time of the Civil War. A disappointingly small number of schools report this aid available, about one school in ten. Of the schools which do not have the aid available, only one in ten desire the aid which can play such a vital part in enriched and more meaningful instruction in plane and solid geometry.

Activities.

In investigating the status of activities in mathematics classrooms and laboratories, an attempt is made to answer two questions:

1. To what extent are the activities listed used in connection with different mathematical subjects?
2. Are these activities extra-curricular only, regularly or sometimes required, encouraged but not required, or not engaged in because of lack of time?

Table XII Percentages of schools using various activities in connection with different mathematical subjects.

Activities	A	B	C	D	E
School Journeys.	4.7	2.7	2.0	3.3	2.7
Construction of Models in and Out of Class.	4.0	3.3	18.7	46.7	9.3
Planning and Working on Mathematics Exhibits	6.7	8.7	8.7	7.3	7.3
Mathematical Games and Contests.	11.3	18.7	10.7	4.0	4.0
Mathematical Plays and Dramas. .	1.3	2.0	2.0	.7	1.3
Mathematical Recreations and Amusements.	6.0	12.7	10.7	5.3	6.7
Mathematics Clubs.	2.0	8.7	2.7	4.0	6.0

A: Arithmetic
B: Algebra
C: Plane Geometry
D: Solid Geometry
E: Trigonometry

Table XII answers the first of these questions. Construction of models appears to be used most frequently in connection with plane and solid geometry. Approximately one half of the schools reporting make use of this aid in connection with solid geometry, and about one-fifth of the schools use it in connection with plane geometry. About one-fifth of the schools report using mathematical games or contests in algebra courses.

A detailed analysis does not appear warranted. However, it should be mentioned that with the exception of the two cases cited, all the remaining activities listed are not used by more than one school in 10 in connection with any mathematical subject.

Table XIII answers the second question. It should be mentioned at this point that the table answers this question only. The facts reported in this table cannot be tied up with those in Table XII, as might be desired. An attempt at correlating the information supplied by the two tables would have resulted in lengthening the check list to the point of limiting the return.

Table XIII Percentages of schools making different uses of activities.

Activities	A	B	C	D	E
School Journeys.	11.3	0	1.3	10.0	28.4
Construction of Models in and Out of Class.	4.0	23.3	14.7	21.3	12.0
Planning and Working on Mathematics Exhibits	5.3	3.3	8.7	9.3	28.7
Mathematical Games and Contests.	5.3	2.0	14.0	8.0	24.7
Mathematical Plays and Dramas. .	5.3	0	3.3	3.3	33.3
Mathematical Recreations and Amusements.	8.0	.7	8.0	16.0	21.3
Mathematics Clubs.	11.3	.7	2.0	8.0	30.7

A: Extra-curricular only
B; Regularly required
C: Sometimes required
D: Encouraged but not required
E: No time for it at all

Table XIII confirms the statement previously made that construction of models, whether on an extra-curricular basis or a regularly required basis, is the most common activity engaged in in mathematics classrooms and laboratories. Only 12 per cent of the secondary schools report having no time for this activity at all.

Comparatively few activities are engaged in on a "regularly required" basis. Inasmuch as it would be almost impossible to determine just what activities are engaged in in a school which reports that activities are carried on on an "extra-curricular" or a "sometimes required" basis, it appears reasonable to assume that secondary schools, on the

whole, engage in very few activities. This assumption is supported by the fact that schools reporting certain activities as "extra-curricular only" or "sometimes required" are also very few in number.

Selected Comments

Why Include Comments.

It might prove interesting and revealing to show just how some mathematics teachers feel about multi-sensory aids, independently of the status of these aids in their respective school systems. Are teachers, on the whole, enthusiastic supporters of multi-sensory aids to learning? Are they skeptical or undecided about the advantages to be derived?

The comments which accompanied many of the completed check lists may supply a partial answer to these questions. Furthermore, the comments may be used to illustrate and correct some of the more common misgivings and misconceptions entertained with respect to multi-sensory aids.

Comments and opinions, to be sure, are very subjective in nature. At times, however, they may supply information which could never be discovered from facts and figures alone. The fact that a school has very few or no multi-sensory aids at all tells us nothing about the hundred and one reasons for that state of affairs. Therefore, the comments made by teachers, in spite of their subjectivity, may yet make a valuable contribution to the present study.

Types of Comments

Against the use of multi-sensory aids.

Most of the comments were on the subject of films for the teaching of mathematics. These comments ranged from extreme skepticism as to their value, to enthusiastic support for their use in the classroom. In quoting these comments, an attempt will be made to place in one group those whose general tenure is the same. All names of places and persons have been purposely omitted. Comments depicting skepticism will be treated first:

> "The mathematics department has viewed quite a few sound films with the purpose of possible purchase. We have not yet seen any of enough value to warrant the extra expense. Most of the films show processes which can be duplicated at the board, with much more value obtained from the development".

> "Visual education aids are rather hard to obtain down here, although complete equipment is available. I personally cannot see any great value in it except for advanced mathematics".

> "At the present time, I see interest in mathematical studies (all of them) at a very low ebb, despite the increasing demands of the colleges for more and more ground to be covered. I have seen some films. They are educative for grownups, but would be mere entertainment for high school "teen-agers".

> "I have yet to see the film strips or slides that are constructively helpful, not merely review or entertainment".

> "Of course, money for education is at a premium here as elsewhere. Yet I am not sure that some of the above (aids) couldn't be obtained (with a fight) if we thought the results were commensurate with the cost. Most of the above, I feel, are interest arousers rather than truly educative aids!"

Critical evaluation of these comments.

The general tenure of the comments quoted is skepticism of the value of films and film-strips in the teaching of mathematics. Regardless of one's feelings in the matter, it is obvious that certain aspects of the comments made could stand investigation and possibly, correction.

In one instance, it is mentioned that most of the mathematics films reviewed by a certain mathematics department showed processes which could be duplicated at the board. If such is the case, the benefit to be derived from that or those films would not warrant the extra expense. The teachers concerned acted wisely in not purchasing or renting such films. However, this should not deter mathematics teachers from attempting to keep abreast of developments in the field of mathematical films. More and more are being produced, although in comparison with films in other fields, mathematics films are not far from the experimental stage. Teachers of mathematics should keep constantly in mind that motion pictures are of greatest benefit, and make their greatest contribution to the educative process where motion is involved. If the process shown on a film could be duplicated on the blackboard, it is a waste of time and money to invest in such films. However, when one considers that in addition to portraying motion, films can attract and hold the attention of pupils

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because of their novelty, and add life and interest to mathematical instruction as well, one will readily see how valuable an aid to instruction films can be.

In another interesting comment, we find expressed the belief that films have no great value except for advanced mathematics. This statement might be said to be partially true and partially false. Mr. Henry W. Syer[1] of Boston University's School of Education, a pioneer and enthusiastic worker in the field of mathematics films gives some suggestions as concerns subjects for films. The mathematics teacher interested in making or using motion pictures would do well to keep these words in mind:

> "The teacher who is alert to such phrases as: "if point P approaches P'," "the area approaches a limit," "the number of divisions becomes infinite," "place AB on A'B'," "under pressure the circle becomes an ellipse," or "the angle increases," will have no scarcity of subjects. In short, by noting the thousand and one places where change of position (motion), change of shape (transformation), correspondence, or limit is implied, topics will be found. The secret is to use motion where it is implied in the mathematical argument, and where the expert and gifted mathematician supplies it with his intuition and imagination".

These words, from an expert in the field, show that mathematics films are not restricted to the field of advanced mathematics.

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1. Henry W. Syer, "Making and Using Motion Pictures for the Teaching of Mathematics", Multi-sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College Columbia University, New York, 1945, p.326.

One teacher is inclined to feel that motion pictures "would be mere entertainment for high school teen-agers". To say that motion pictures may in some cases and for some pupils be mere entertainment is one thing, but to condemn them all as being "mere entertainment" and to overlook the specific advantages is another matter. Whether a film turns out to be mere entertainment or not depends to a large extent upon the teacher. Wise initial selection of a film, by the teacher, with a follow-up of discussion, testing, and reports by the pupils after the film has been shown will soon cause a class to look upon films as instructional rather than entertaining.

Still another teacher voices the opinion that films and film slides are not constructively helpful, but merely review or entertainment. In the previous paragraph, we have treated the question of entertainment. The question of "review" can be disposed of quickly. There could be little disagreement with the opinion expressed that films and film slides are excellent instruments for review of mathematical facts and concepts. However, it is not necessary to prove that a good review is constructively helpful, a statement which by many teachers might be considered as bordering on the axiomatic. Thus, while seemingly skeptical of the value of films and film slides, that teacher is actually voicing one of the strongest arguments for the use of these aids.

Most aids to learning "are interest arousers rather than truly educative aids". This opinion, falls in the same category as the opinion expressed in the previous paragraph. A strong argument <u>for</u> the use of multi-sensory aids <u>is</u> the fact that the pupils' interest is aroused to a greater extent than could ever be accomplished by the traditional method of instruction. Is an aid which arouses interest an educative aid? The positive answer to this question can be found in almost any text on educational theory and principles of learning.

<u>Conclusions</u>.

In the preceding paragraphs have been discussed the opinions of some teachers who question whether films, filmstrips, and slides make any significant contribution to the educative process. These opinions have helped to reveal some of the shortcomings of these aids. Such opinions are valuable and desirable in that they reveal areas where improvements are needed. They also show that many teachers are not prone to accept at face value all claims made for multi-sensory aids. This characteristic is desirable, but is also one which, if carried too far by too many teachers would result in stagnation of teaching procedures. The conscientious teacher at all times keeps an open mind toward suggestions for the improvement of his or her teaching procedures.

A disconcerting revelation, however, is the fact that some of the comments made by teachers reveal an inadequate understanding of principles of learning and educational theory. It has been wisely said that 19 years of teaching experience may be in truth but one year of experience repeated 19 times. The failing in this case can be corrected by keeping abreast of developments in the field of education. The conscientious teacher, just as the conscientious doctor, cannot fail to do this and retain his or her professional standing. Have teachers of mathematics been as responsive to changing educational theory and as receptive of aids to the improvement of instruction as teachers in other fields? Perhaps these two questions which seem inextricably bound together may be answered when the primary objective of the present study has been achieved.

<u>For multi-sensory aids or undecided</u>.

After studying carefully the remaining comments, the writer came to the conclusion that little was to be gained by a further breakdown in types of comments. All remaining comments will be treated in the following paragraphs. The treatment will be the same as in the preceding section. The comments will first be quoted, and then discussed.

> "In our small high school, (6 teachers, 80 pupils), we are not in a position financially to purchase films nor to rent any save the least expensive. A heavy and varied teaching load prevents the other mathematics teachers and myself from promoting the various mathematical activities listed".

"I would take great pleasure in making use of these good suggestions if it could be arranged that the students would not miss the important and workable part of their high school math; but I find that I have hardly enough time in class to prepare them adequately for the minimum requirements of the College Entrance Board. In a five year high school, I believe these extras would be excellent".

"We use many sound films in our vocational guidance department to show occupations and vocations that necessitate a mathematical background.

The film, "Property Taxation" has been used by us with great success in our Economics class in the study of taxation".

Note: The writer has viewed the above-mentioned film and considers it excellent for a class in Social Mathematics or mathematics for everyday life.

"Our time in the classroom is so short that even now, without visual aids, we hardly have time to cover the required work. I believe it would be fine to use them for extra-curricular activities and I wish we had some of the films listed available. However, I still belong to the "old school" which believes that drill work is of prime importance. Do you think enough is gained from the visual aids to repay the time lost in classwork? However, I would be willing to give it a try".

"We would like to have all these aids at our disposal, but we are financially poor".

"In a small school such as ours, we find it too costly to purchase our films. We do, however, make use of several films which are obtained from the various visual aid concerns on a rental basis".

"I consider mathematics clubs and mathematics plays very valuable, but with six classes a day and four preparations, I just don't try much of that sort".

"At __________, movies are encouraged in all subjects, but the reels are all rented. None are kept in stock. All movies are held in one room.

Little is done in the field of mathematics concerning recreation. This is due to the fact that the school has many other extra-curricular activities".

"Lack of funds seems to hinder obtaining the aids you have listed".

"The use of visual and audio-visual aids is much in favor among the teachers at ______. We expect to incorporate their use more regularly as conditions permit in the regular curriculum.

At present, no funds have been made regularly available for purchase or rental of audio-visual aids such as films, but we hope to improve this situation in the future".

"A summary of your report will be appreciated. I have used films in the past, but have not been entirely satisfied with them. At present, I am too busy to make a careful check of all the newer ones, including strip films. A report on these will be of help in scheduling those that are worthwhile".

"I wish some clearning house were known and available to teachers of Mathematics, where information as to visual aids, films, prices, etc. would be obtainable, and whereby some description or evaluation of these aids was obtainable. Where do the films you have listed come from, and how expensive are they?"

"Two basic handicaps to our use of multi-sensory aids exist:

1. Lack of wiring, equipment, shades, etc..
2. No rooms assigned to me as a math room so I am a trespasser on the space of another teacher and have to remove all my belongings at the end of each period. This makes exhibits, etc. impossible".

"Much as I would like to have these valuable aids, our classes are much too small to warrant the expenditure. For example, in trigonometry there are three pupils; advanced math, two; plane geometry, eight; algebra, five."

"We are very much interested in the motion picture as an aid in teaching mathematics, and I hope to have some films very soon".

Causes of Indecision

Lack of time.

The factors which contribute to indecision as regards to the use of multi-sensory aids and the benefits to be derived from the use of these aids may be outlined as follows:

1. Lack of time.
2. Lack of money.
3. Lack of information concerning aids to teaching.
4. Lack of facilities for use of some aids.

Of these four, the most common is lack of time. Mathematics teachers are kept so busy preparing students for College Entrance Exams that they have little or no time to investigate or use certain multi-sensory aids or "extras" as some would call them. Is this a valid excuse? Can one assume that teachers of mathematics who give this excuse have carefully studied the College Entrance Requirements in their subject field?

Let us look briefly into the matter of College Entrance Requirements in general and for Mathematics in particular. Brammell's[1] study give us all the information we need. In general, Brammell discovered that comparatively

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1. P. Roy Brammel, "Articulation of High School and College," Bulletin, 1932, no. 17, National Survey of Secondary Education, Monograph, no. 16 (Washington, D.C.) United States Office of Education.

few higher institutions of learning either recognize or require the examination of the College Entrance Examination Board[1]. Furthermore, the examination of the College Entrance Examination Board was discovered to be but one of twenty-one different avenues of college entrance[2]. Even in such college departments as science and engineering which can reasonably be expected to require more mathematics than many others, the requirements ranged all the way from none to four and one half Carnegie units[3].

In view of this information, can a mathematics teacher reasonably offer lack of time as an excuse for not attempting to enrich instruction through the use of multi-sensory aids?

Lack of money.

Another prominent excuse is lack of money. This is obviously directed at the cost of projection equipment and materials, and is a much more valid excuse than lack of time. However, a Research Bulletin[4] of the National Education Association may help to throw some light on this subject.

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1. Ibid., p. 13
2. Ibid., p. 12
3. Ibid., p. 36
4. National Education Association, "Audio-Visual Education in City School Systems," Research Bulletin, 24:165, December, 1946.

The following table is adapted from this Research Bulletin:

Per pupil expenditure on audio-visual education 1945-1946, in various groups of city school systems with audio-visual departments.

Population	Over 100,000	30,000 to 100,000	10,000 to 30,000	5,000 to 10,000	2,500 to 5,000	Ave.
Per Pupil Expenditure	$ .32	$ .57	$ .83	$ .89	$1.68	$.86

The above figures clearly indicate that the cost of maintaining audio-visual departments is not prohibitive. However, in the last analysis, it is up to the individual school systems to decide what can be set aside for the purchase and renting of multi-sensory aids. Much can be done if money is available. Paradoxical as it may seem, however, a great deal can be done with little or no money at all.

A wealth of pictorial material is obtainable literally for the taking in the countless popular magazines, pamphlets, advertisements, and booklets so readily accessible to all. Life and Fortune magazines are particularly fertile sources of pictures which can be used in mathematics classrooms or laboratories; as pictures illustrating practical applications of mathematics in every day life; geometry in architecture; higher mathematics in radio and electronic devices. The resourceful and imaginative teacher can guide the pupils in the collection, mounting, and preservation of pictures particularly pertinent to certain aspects or topics in mathematics.

Useful models can be made by pupils with an inclination toward this type of work at a very small cost. These models can be made from cardboard, wire, clay, balsa wood, celluloid, or any combination of these. The best models can be preserved along with the mounted pictures previously mentioned.

In every group, there are bound to be several individuals interested in photography. Under the teacher's direction, such individuals would doubtless delight in taking pictures to be converted into photographic lantern slides. Pupils interested in typing and drawing would need little coaxing to produce cellophane and etched glass slides as their contribution to the class stock of multi-sensory aids. If the work of making lantern slides is carefully planned, and objects for each slide are carefully chosen to minimize waste, the cost per slide will average between ten and fifteen cents.

The attempt has been made to show that a considerable headway can be made in initiating and conducting a program of multi-sensory activities at a very small cost. The reader is referred back to Chapter II where multi-sensory aids available at little or not cost have already been taken up in greater detail. Lack of money is not a valid excuse for neglecting the possibilities in multi-sensory aids when teacher resourcefulness, imagination, and hard work, combined with the almost unlimited potential energy of a group of healthy high school youngsters can do so much to improve,

enrich, and make instruction in mathematics more enjoyable to all concerned.

Lack of information concerning aids to teaching.

Mathematics has been taught in the traditional way for so long that unusual or experimental work in this field is the exception rather than the rule. For this same reason, little use has been made of multi-sensory aids by mathematics teachers as a whole, However, while not as extensive as that in other fields, information on aids to the teaching of mathematics is available. Two periodicals in particular: The Mathematics Teacher[1] and School Science and Mathematics[2] make it a point to publicize experimental work on the improvement of instruction in mathematics. Many are the articles to be found in these periodicals on the use of aids to learning by resourceful and progressive teachers. Many of these articles can be found listed in the supplementary bibliography in the Appendix.

Much information is to be obtained from commercial houses which manufacture films, projectors, instruments and other aids to learning, and from distributors of these aids. For this reason, there has been included in the Appendix of the present study a selected list of "Sources" where the multi-sensory aids discussed in the study as well as

1. The Mathematics Teacher, 525 West 120th Street, New York 27, New York.

2. School Science and Mathematics, 450 Ahnaip Street, Menasha, Wisconsin.

information concerning their use can be obtained. The mathematics teacher truly desirous of improving his or her teaching procedure can findmuch in the above-mentioned sources to help achieve this goal of improved and enriched teaching.

Lack of facilities for use of some aids.

A partial answer to those teachers complaining of lack of facilities for the use of certain aids, particularly films and other aids requiring projection equipment has been given in the pages dealing with cost. There are, however, certain implications which cannot be answered so easily.

In the ideal situation, unlike the present situation in which a room may be used by a class in English one period then by a class in French, etc., a room is set aside for classes in Mathematics only. This room will be equipped with opaque shades, a portable or permanent motion picture screen, and adequate electrical wiring facilities for the use of projection equipment.

"Unfortunately, the planning and building of schools in the past has been done more as an emergency requirement than as a carefully considered educational plan checked and rechecked by present and estimated future needs and by current and future practices as revealed through trends[1]."

1. Leonard V. Koos, James M. Hughes, Percival W. Hutson, and William C. Reavis, Administering the Secondary School, American Book Company, New York, 1940, p.505.

In the case of large high schools, even if not carefully planned at the time of their construction, the ideal situation as outlined in a previous paragraph can be approximated. Such is hardly the case for the small high school. The logical answer to this problem appears to be the present trend toward consolidation of several small schools into one large system.

The duty of those concerned with the planning of new educational buildings is obvious. In weighing future needs and future practices, the best interests of pupils demand that adequate space provision and facilities be made available to teachers desirous of incorporating multi-sensory aids into their teaching procedure.

Chapter IV

SUMMARY AND SUGGESTIONS
FOR FURTHER STUDIES

Present Status of Multi-Sensory Aids

Blackboards and blackboard equipment.

With the exception of pantographs and spherical blackboards, schools are reasonably well equipped with these aids, but do not use them as frequently as should be expected in mathematics classrooms and laboratories. Approximately 82 per cent of the schools contacted never use pantographs, and approximately 67 per cent never use spherical blackboards. More extensive use than is at present indicated could also be made of the other aids listed.

Still pictorial materials.

The study reveals that mathematics classrooms and laboratories are not nearly as well equipped with this type of material as they should be. Particularly is this deplorable when one considers the ease with which this type of material can be obtained. The percentages of schools which never use this type of aid in connection with the teaching of mathematics range from 28.7 per cent for graphs to 85.4 per cent for wall charts. Such a state of affairs does not speak well for teachers of secondary mathematics in Massachusetts.

Laboratory equipment.

The status of laboratory equipment is obviously not what it should be. Entirely too many schools never make use of any of the equipment listed. For example, nine out of every ten schools never make use of calculating machines, filing cabinets, or exhibit cases. This situation must be remedied. It is deplorable not only in itself, but is a drawback to the use of still pictorial materials and the planning of exhibits.

Projection facilities.

The present study reveals that no more than 50 per cent of the schools at present have mathematics classrooms adequately equipped, insofar as electrical outlets, screens, and darkening facilities are concerned, to make use of projection equipment.

Projection equipment.

The schools appear fairly well equipped with motion picture projectors with approximately 71 per cent of the schools reporting this aid on the premises. Whether the mathematics departments of all these schools have easy access to this equipment is not revealed by the present study. The schools are not so well equipped with opaque or lantern slide projectors. Approximately 50 per cent report the presence of lantern slide projectors, and slightly more than 25 per cent report that of opaque projectors. On the whole, the possibilities of making extensive use of projected aids in mathematics departments appear dim for some time to come.

The existing deficiencies are not of the type which can be corrected immediately.

Projected aids.

The present status of projected aids is such that films, film strips, and lantern slides are not available in approximately 9 out of every 10 schools. Of those schools which do not have these aids, approximately two-thirds indicate no desire to have these aids available. A summary of teacher comments indicates that some teachers are deterred in the use of these aids by lack of time, lack of money, lack of facilities for projection, lack of information, and skepticism as to the benefits to be derived.

Activities.

In general, it would appear that construction of models in connection with plane and solid geometry is the activity most frequently engaged in on a required basis. The other activities listed are engaged in by very few schools and on a basis other than "required". On the whole, very little is done in this area by the mathematics departments of Massachusetts secondary schools.

Suggestions for further studies.

Much is still to be done in the field of multi-sensory aids for the teaching of mathematics. Persons contemplating studies in this field might consider the following suggestions:

1. Controlled studies to prove or disprove the benefits claimed for extensive use of multi-sensory aids as opposed to the traditional method of instruction.

2. Studies to determine the minimum cost of incorporating adequate "multi-sensory" programs in the mathematics departments of schools of different sizes.

3. Studies on the relative effectiveness of different multi-sensory aids for teaching different mathematical concepts.

4. Studies or actual reports on mathematics departments making extensive use of multi-sensory aids.

5. Service papers on units in mathematics which make definite provision for the use of different multi-sensory aids.

On the whole, the mathematics departments of Massachusetts make disappointingly small use of multi-sensory aids.

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Johnson, Donovan A., "Vitalizing Geometry for the Use of Pictures", School Science and Mathematics, 38:1032-1034, (December, 1938).

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Parsons, Ada N., "Colored Crayons as an Aid in Teaching Mathematics", School Science and Mathematics, 14:33-35, (January, 1914).

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Read, Cecil B., "Mathematical Magic", School Science and Mathematics, 37:597, (May, 1937). Also, 37:650, (June, 1937).

Reeve, W.D., "The Tree of Knowledge", Mathematics Teacher, 28:56, (January, 1935).

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Smith, Jessie Roselle, "A Mathematics Work Room for the Senior High School", Mathematics Teacher, 38:126-129, (March, 1945).

Smith, Alice K., "The Case of Matthew Mattix", Mathematics Teacher, 36:286-291, (May, 1933).

Spenser, M.C., "A Color Scheme for Figures in Solid Geometry", School Science and Mathematics, 10:549, (June, 1910).

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Starr, David W., "Enrichment Materials for First Year Algebra", Mathematics Teacher, 32:68-77, (February, 1939).

Starr, Oscar L., "A Projectile Experiment", School Science and Mathematics, 33:956-958, (December, 1933).

Vaughn, Adah, "Professor Whiz and His Class in Magic Math", School Science and Mathematics, 39:540-545, (June, 1939).

Volpel, Marvin C., "Solid Geometry Made More Interesting", School Science and Mathematics, 38:740-742, (October, 1938).

Waters, Idella, "Vitalizing Geometry Through Illustrative Material", Mathematics Teacher, 28:101-110, (February, 1935).

Watson, E.E., "Our Geometric Environment", School Science and Mathematics, 39:258-259, (March, 1939).

Weinter, N., "From Interest to Interest", Mathematics Teacher, 30:23-26, (January, 1937).

Woodford, E.B., "Teaching the Locus Concept in Plane Geometry", Mathematics Teacher, 26:105-106, (February, 1933).

Woolsey, George, "Measuring to a Billionth of an Inch with a Pocket Handkerchief", School Science and Mathematics, 38:280-281, (March, 1938).

Yates, Robert C., "An Ellipsograph", National Mathematical Magazine, 12:213-215, (February, 1938).

Yates, Robert C., "The Story of the Parallelogram", Mathematics Teacher, 33:301-310, (November, 1940).

Yates, Robert C., "To Have and to Hold",(an experiment), National Mathematical Magazine, 14:2-4, (October, 1939).

A P P E N D I X

SELECTED SOURCES OF MULTI-SENSORY AIDS

Blackboards and Blackboard Equipment

1. Eugene Dietzgen Company
   116 West Monroe Street
   Chicago, Illinois

2. J.L. Hammett Company
   Kendall Square
   Cambridge 42, Massachusetts

3. Keuffel & Esser Company
   127 Fulton Street
   New York, New York

4. Yoder Instruments
   East Palestine, Ohio

Bulletin Boards

Sources #2 and #4

Maps

5. Superintendent of Documents
   Washington, D.C.
   (Price List 53 Maps, September, 1940. Maps of all sorts at nominal prices).

Photos of Mathematicians

6. Scripta Mathematica
   Amesterdam Avenue & 186th Street
   New York, New York

7. Stanley Bowmar Company
   2929 Broadway
   New York, New York

Construction Paper, Cardboard, Etc.

Sources #2, #3, and #4.

Wall Charts

8. Museum of Science and Industry
   Jackson Park
   Chicago, Illinois

Also source #5.

Calculating Machines

9. Burroughs Adding Machine Company
40 Broad Street
Boston, Massachusetts

10. Comptometer Company
52 Chauncy Street
Boston, Massachusetts

11. Monroe Calculating Machine Company
683 Boylston Street
Boston, Massachusetts

12. Remington Rand Inc.
857 Commonwealth Avenue
Boston, Massachusetts

Filing Cabinets

Sources #2 and #12.

Exhibit Cases

13. Central Scientific Company
1700 Irving Park Boulevard
Chicago, Illinois

Lantern Slides

14. Keystone View Company
Meadville, Pennsylvania

Projection Equipment

15. Ampro Corporation
2835 N. Western Avenue
Chicago 18, Illinois

16. Bausch & Lomb Optical Company
Rochester, New York

17. Bell & Howell
30 Rockefeller Plaza
New York, New York

18. Eastman Kodak Company
Rochester, New York

19. Spencer Lens Company
Rochester, New York

Solid Geometry Models

Source #4.

Slide Rules

Sources #1, #2, #3, and #4.

Motion Pictures

20. Bald Eagle Film Productions
104 Havre Street
New Haven, Connecticut

21. Encyclopaedia Britannica Films
20 North Wacker Drive
Chicago, Illinois

22. Expanding Cinema
422 West 46th Street
New York, New York

23. Ideal Pictures Corporation
28 East Eighth Street
Chicago 5, Illinois

24. Knowledge Builders
625 Madison Avenue
New York, New York

25. Society for Visual Education
100 East Ohio Street
Chicago, Illinois

26. United World Films
30 Rockefeller Plaza
New York, New York

27. Visual Education Service
116 Newbury Street
Boston, Massachusetts

28. Young America Films
32 East 57th Street
New York, New York

29. Y.M.C.A. National Board
347 Madison Avenue
New York, New York

Film Strips

30. Cambosco Scientific Company
37 Antwerp Street
Allston, Massachusetts

31. Curriculum Films
Americolor Service
R.K.O.Building
Radio City
New York, New York

32. Jam Handy Organization
2900 East Grand Boulevard
Detroit, Michigan

Also sources #25 and #27.

Stereographs

33. Keystone View Company
Meadville, Pennsylvania

Also source #25.

BOSTON UNIVERSITY

School of Education

Questionnaire on the Present Status of Aids to Learning in the Mathematics Classrooms and Laboratories

Please circle the number at the left of those aids which you have, and check the proper column to indicate the frequency with which each is used(where indication of frequency is appropriate.)

	often	seldom	never
1. Blackboards (squared)			
2. (spherical)			
3. Blackboard rulers			
4. Blackboard compass			
5. Blackboard protractors			
6. Colored chalk			
7. Pantographs			
8. Bulletin boards			
9. Maps			
10. Graphs			
11. Diagrams and posters			
12. Cartoons			
13. Photos of mathematicians			
14. Construction paper, cardboard, etc.			
15. Wall charts			
(Please list names of charts			
16. Calculating machines			
Please list any others.			
17. Filing cabinets			
18 Exhibit cases			
19. Solid geometry models			
20. Slide rules(large demonstration type)			
21. Slide rules (for individual use)			
22. Book cases and shelves			
23. Mathematics books for use by the pupils, other than the regular text.			

24. Are your mathematics classrooms equipped with electrical outlets? Circle: Yes No

25. Are your mathematics classrooms equipped with motion picture screens, either stationary or portable?
Circle: Yes No

26. Can your mathematics classrooms be darkened for visual aids?
Circle: Yes No

The columns A, B, C, D and E represent the following:

A: Available in room at all times.
B: Available in school upon request.
C: Available in school system upon requisition.
D: Not available.
E: Should like to have the aid available.

Please place a checkmark in the appropriate column or columns.

	A	B	C	D	E
27. Opaque projector					
28. Lantern slide projector					
29. Lantern Slides:					
30. Plain glass					
31. Ground glass					
32. Cellophane					
33. Silhouette					
34. Photographic					
35. Kodachrome					
36. Sound film projector					
37. Sound films:					
38. 'Origin of mathematics'					
39. 'Geometry in action'					
40. 'Rectilinear coordinates'					
41. 'The slide rule'					
42. 'Precisely so'					
43. 'Locus'					
44. 'Lines and angles'					
45. 'Angles'					
46. 'The circle'					
47. 'Chords and tangents of circles					
48. 'Angles and areas in circles'					
49. 'Vernier scale'					
50. 'Long division'					
51. 'Property taxation'					
52. 'What is four?'					
Please list others:					
53. Silent films:					
54. 'Frequency curves'					
55. 'The isograph'					
56. 'Our children's money'					
57. 'Einstein's theory of relativity'.					
Please list others.					
58. Film Strips:					
59. 'The origin of algebra'					
60. 'Basic definition of algebra'					
61. 'Introduction to plane geometry'					
62. 'The circle'					
63. 'Angular Measurement'					
64. 'Constructions'					
65. 'Positive and negative numbers'					
66. 'Ratio and proportion'					
67. 'Exponents and logarithms'					

	A	B	C	D	E
68. 'Rectilinear figures'					
69. 'Math instruments series' (5 strips)					
70. 'Plane geometry (color 16 strips)'					
Please list others.					
71. Stereographs					

Activities Please place a check in the appropriate columns.	Used in connection with: Arithmetic	Algebra	Plane geometry	Solid geometry	Trigonometry	Type of use: Extra-Curricular only	Regularly required.	Sometimes required.	Encouraged, but not required.	No time for it at all.
72. School journeys										
73. Construction of models, in and out of class										
74. Planning and working on mathematics exhibits.										
75. Mathematical games and contests										
76. Mathematical plays and dramas										
77. Mathematical recreations and amusements										
78. Mathematics clubs										

Your suggestions, comments, and criticisims will be gratefully received. Please utilize the attached sheet for this purpose.

FILMS AND FILM STRIPS FOR THE TEACHING OF MATHEMATICS

Henry W. Syer

Boston University - School of Education

Boston, Massachusetts

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Part I Sources

1. Akin and Bagshaw, Inc.
1425 Williams Street
Denver, Colorado

2. American Films Foundation, Inc.
542 Fifth Avenue
New York City

3. Art Film Company
Boston, Massachusetts

4. Bald Eagle Film Productions
104 Howe Street - Annex
New Haven, Connecticut

5. Baldus, R.
Munchen
Bavaria, Germany

6. Bell & Howell Co.
1801-1815 Larchmont Avenue
Chicago, Illinois

7. Boyd, Rutherford
112 Prospect Street
Leonia, New Jersey

8. Bray Rental Library
330 West 42nd Street
New York City

9. Bruce, Claribel E.,Principal
#52 School, 100 Farmington Road
Rochester, New York

10. Cambosco Scientific Co.
37 Antwerp Street
Allston, Mass.

11. Carter Cinema Producing Corp.
c/o Evans Film Laboratories
1476 Broadway
New York City

12. Castle Films Inc.
30 Rockefeller Plaza
New York City

13. College Film Center
59 East Van Buren Street
Chicago, Illinois

14. Coronet Films,
Glenview,
Illinois

15. Curriculum Films,
Americolor Service
RKO Building - Radio City
New York City

16. Dance - Kaufman
18 Upper Stanhope Street
Liverpool 8, England

17. DeLuxe Laboratories
New York City

18. Department of Education
Columbus,
Ohio

19. Dynamic Pictures
729 Seventh Avenue
New York City

20. Eastman Kodak Co.
Teaching Films Division
Rochester, New York

21. Edited Pictures System, Inc.
330 West 42nd Street
New York City

22. Education and General Service Ltd.
37 Golden Square
London, England

23. Elgin National Watch Co.
Elgin,
Illinois

24. Encyclopedia Britannica Films
20 North Wacker Drive
Chicago, Illinois

25. Expanding Cinema
422 West 46th Street
New York City

26. Film Centre
34 Soho Square
London, W. 1, England

27. Filmette Films

28. Films Inc.
330 West 42nd Street
New York City

29. Ford Dealer - Local

30. General Electric Company
Visual Instruction Section
1 River Road
Schenectady, New York

31. General Motors Corporation
Dept. of Public Relations
3044 West Grand Boulevard
Detroit, Michigan

32. Greiner, A.
Berlin, Germany

33. Grokarick, A.J.
Eldridge Park,
Trenton, New Jersey

34. Heinrich, H.
Breslau, Germany

35. Hildebrandt, Dr. E.H.C.
Northwestern University
211 Lunt Building
Department of Mathematics
Evanston, Illinois

36. Hopper, Grace
Vassar College
Poughkeepsie, New York

37. Jam Handy Organization
2900 E. Grand Boulevard
Detroit, Michigan

38. Keniston, Rachel
1202 N. Hunter Street
Stockton, California

39. Knowledge Builders
625 Madison Avenue
New York 22, New York

40. Massachusetts Institute of
Technology
Cambridge, Massachusetts

41. Montelbano

42. National Film Library
British Film Institute
4 Great Russell Street
London, W.C. 1, England

43. Pathescope Library
438 Stuart Street
Boston, Massachusetts

44. Nearest office of
United States Sedret Service

45. Offhouse, Charles D.
Eastside High School
Paterson, New Jersey

46. Prudence Advertising Co.
331 Madison Avenue
New York City

47. R.C.A. Laboratories
Camden, New Jersey

48. Smith, Miss L.E. (Librarian)
Bell Telegraph Laboratories
463 West Street
New York City

49. Society for Visual Education
327 South La Salle Street
Chicago, Illinois

50. Teaching Film Custodians
25 West 43rd Street
New York City

51. United States Army Films

52. University of Colorado
Boulder, Colorado

53. University Extension
State Division
Boston, Massachusetts

54. University of New Hampshire
Durham,
New Hampshire

55. University of Wisconsin
Bureau of Visual Instruction
Madison, Wisconsin

56. Vibbins, Arthur
Darien,
Connecticut

57. Visual Education, Ltd.
31 St. Martin's Lane
London, W.C.2, England

58. Visual Education Service
131 Clarendon Street
Boston, Massachusetts

59. Visual Sciences
Suffern,
New York

60. Whitman, Prof. E. A.
Carnegie Institute of Technology
Pittsburgh, Pennsylvania

61. Wholesome Film Service Inc.
48 Melrose Street
Boston, Massachusetts

62. Y.M.C.A. National Board
347 Madison Avenue
New York City

63. Young America Films
32 East 57th Street
New York 22, New York

Film	Sources
1. Abstractions (Visual Music)	58
2. Algebra	45
3. Angle Sum of a Triangle	35
4. Angles	39
5. Angles and Arcs in Circles	39
6. Ann Faces the Future (G.V.)	62
7. Arthur and Dad Come Home by Train	41
8. Big Money	62
9. Bridges	1
10. Cardiods	36
11. Chords and Tangents of Circles	39
12. Churches and Cathedrals I and II	58
13. The Circle	39
14. The Circle	41
15. Circles	9
16. Community Patterns in Geometry	56
17. Concurrency in Triangles	38
18. Cone and Cylinder	41
19. Congruent Figures	39
20. Creative Design in Painting	18
21. Crystallization	58
22. Cube and Square Root	11
23. Cubes of Constant Width	35
24. Definition of Plane Geometry	3
25. Development of Transportation	24
26. Dewfall - Natures Jewel	55
27. Einstein's Theory of Relativity	55
28. Equation $\ddot{x} + x = 0$	42
29. Equation $\ddot{x} + x = A \text{ Sin } Nt$	42
30. Euclid I.32 (Angle Sum of Triangles)	42
31. Federal Reserve System	62
32. Fixed Gages	12
33. For All Eternity	62
34. Frequency Curves (Wis)	55
35. The Generation of Involute Gear Teeth	35
36. Geometry	53
37. Geometry (Sampson)	61
38. Geometry in Action	4
39. Getting Your Money's Worth	13
40. Gravitation - The Moon - Constellations	21
41. Height Gages and Standard Indicators	12
42. How Our Eyes Deceive Us	43
43. Hypocyclic Motion	35
44. Individual Differences in Arithmetic	24
45. Introduction to Mechanical Drawing	52
46. Isograph	48
47. Know Your Money	44
48. Latitude and Longitude	21
49. Lines and Angles	39
50. Locus	39

	Film	Sources
51.	Locus In Plane Geometry	38
52.	Long Division	24
53.	The Lord Helps Those - Who Help Each Other	62
54.	Making Money	62
55.	Measurements and Life	41
56,	Mensuration	57
57.	Micrometer	12
58.	Modern Banking	21
59.	Modes and Motors	31
60.	A Money Making Industry	62
61.	Mouvements Vibratoires	26
62.	Mrs. Brown Goes Shopping	41
63.	Mysteries of Snow	57
64.	Nation's Market Place	19
65.	Now for Tomorrow	62
66.	Oh Say Can You See (on taxes)	2
67.	Origin of Mathematics	
68.	Our Children's Money	62
69.	Parabola	7
70.	Play of Imagination in Geometry, The	24
71.	Precisely So	31
72.	Prisms and Pyramids	41
73.	Propagation of Waves	16
74.	Property Taxation.	24
75.	Protecting the Consumer.	
76.	Q.R.S.	
77.	Quadrilaterals.	39
78.	Rate of Change	35
79.	Rectangle Ramily	41
80.	Rectilinear Coordinates	6
81.	Rectilinear Coordinates	39
82.	Resultant Circle and Straight Lines	16
83.	Resultant Ellipses	16
84.	Rhythm in Light	25
85.	Rotation and Revolution	21
86.	Royal Mint	62
87.	The Savings Bank	41
88.	Science Rules the Rouge	29
89.	The Skilled Mechanic	21
90.	Social Security Benefits	17
91.	Sphere and Hemisphere	41
92.	Steel Rule	12
93.	Synchromy No.2	25
94.	Teaching Creative Design	62
95.	Telescope Making in the West Allis, Wisconsin High School	55
96.	Theorem of Pythagoras	42
97.	Thrift	62
98.	Transportation	28
99.	Triple Ingegral	60
100	Velocity	

Film	Sources
101. Vernier Scale	12
102. We Are All Artists	58
103. What Is Four?	63
104. When You Can Measure	30
105. Workers' Old Age and Survivors Insurance	

Part III Mathematics Film-Strips

Film	Sources
1. Addition and Subtraction (Arithmetic).	37
2. Addition and Subtraction of Fractions.	37
3. Addition and Sbutraction in Geometry	37
4. Analytic Geometry	37
5. Angular Measurement.	37
6. Applying Geometric Logic-Deductive Reasoning	15
7. Applying Geometric Logic-Definitions & Key Words . .	15
8. Applying Geometric Logic . . . . (Induction, Analysis, and Indirect Reasoning)	15
9. Applying Geometric Logic (Mistakes in Thinking). . .	15
10. Areas and Perimeters (Arith.).	10
11. The Arithemetic of Algebra	37
12. Arleigh's Fractions	49
13. Circles-Chords and Areas (Geometry).	10
14. Circle - Measurement of Angles	10
15. Circle - Tangents and Secants	10
16. Common Denominator (Arith.)	10
17. Constructions (Geometry)	37
18. Decimals (Arith.).	10
19. Differences (Arith.)	10
20. Earth in Motion (Wis.)	55
21. Equations and Formulas (Algebra)	37
22. Exploring the Universe . (Ves)	58
23. Exponents and Logs	37
24. Five Keys to Mathematics	37
25. Foundations of Geometry	15
26. Foundations of Geometry - Postulates: Triangles . . .	15
27. Fourths (Arith.)	10
28. Fractions, Decimals, Percentage (Arith.)	37
29. Geometry in Art	15
30. Geometry	49
31. Geometry in the Home	
32. Geometry in Nature	
33. Geometry Slides in Nature and Architecture.	
34. Graphs (Arith.)	10
35. Graph Uses	37

Part III Mathematics Film-Stiprs - Cont. Page 7.

		Sources
36.	Halves (Arith.)	10
37.	The History of the Measurement of Length	
38.	Instruments (meas. in lab.)	49
39.	Instruments (meas. outside the lab)	49
40.	Instruments (calculation instruments)	49
41.	Instruments (calculation instruments-early methods)	49
42.	Instruments (calculation instruments-advanced methods)	49
43.	Introduction to Plane Geometry	15
44.	Introduction to Plane Geometry (Geom)	10 & 49
45.	Is Seeing Always Believing	
46.	Locus	15
47.	Mechanical Movements	59
48.	Multiplication and Division (Arith.)	37
49.	Multiplication and Division of Fractions	37
50.	Multiplication and Division in Geometry	37
51.	Optical Illusions	59
52.	Optical Illusions	10
53.	Order of Operations (Arith.)	37
54.	Plotting Graphs	37
55.	Positive and Negative Numbers	37
56.	Problem Analysis	37
57.	Problems in Games (Arith.)	10
58.	Products (Arith.)	10
59.	Quotients (Arith.)	10
60.	Ratios and Proportions (Algebra)	37
61.	Rectilinear Figures (Angles and Triangles)	10
62.	Rectilinear Figures (Polygons & Locus of a Point)	10
63.	Rectilinear Figures (Quadrilaterals & Polygons)	10
64.	Rectilinear Figures (Triangles and parallel lines)	10
65.	Rectilinear Figures (triangles)	10
66.	Reduction (Arith.)	10
67.	Scales and Models (Geometry)	37
68.	Square Root and Cube Root (Arith)	37
69.	Thirds (Arith.)	10
70.	Trigonometry	37
71	Vectors (Geometry)	37
72.	Vocabulary (Circle I)	15
73.	Vocabulary (Circle II)	15
74.	Vocabulary (Lines and Angles I)	15
75.	Vocabulary (Lines and Angles II-Relationships)	15
76.	Vocabulary (Lines)	15
77.	Vocabulary (Polygons)	15
78.	Vocabulary (Triangles)	15
79.	Volumes (Arith)	10